جامعة محمد الخامس بالرباط
Université Mohammed V de Rabat

Centre National pour la Recherche Scientifique et Technique

Université Mohammed V
Faculté des Sciences
Rabat

Abstract Booklet

International Days of Mathematics, Statistics and Applications

Journées Internationales de Mathématiques, Statistique et Applications

JIMSA'2025

May 27-29, 2025. Faculty of Sciences, Mohammed V University in Rabat (Morocco)

Organized by the Laboratory of Mathematics, Statistics and Applications (LMSA)

Contact

Pr. Abdelhak Zoglat (Chair) a.zoglat@um5r.ac.ma
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Satellite Event: Training School
Master in *Statistics & Econometrics*
***May 26 & 27, 2025:* A course on non-life insurance**

Book of Abstracts

JIMSA'2025

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*It is with great pleasure and enthusiasm that we welcome you to the **JIMSA'2025** Conference to be held at the Faculty of Sciences of Rabat, Mohammed V University, from May 27 to 29, 2025.*

*After a period marked by various challenges and transformations in the academic and scientific world, this edition of **JIMSA'2025** takes on a special meaning: it is a celebration of reconnection, exchange, and the vitality of our scientific community. The 2025 edition promises to be rich in high-quality scientific presentations, carefully reviewed and selected by the Scientific Committee, and enhanced by a variety of thematic and networking sessions.*

We would like to extend our heartfelt thanks to all those who have contributed to making this event possible:

- *the members of the Scientific, Organizing, and Junior Committees,*
- *the teams responsible for logistics, computing, and technical support,*
- *the Faculty of Sciences of Rabat and Mohammed V University,*
- *as well as our sponsors, in particular the Master's Program in Statistics and Econometrics and the CNRST.*

A special thanks goes to Professor Z. Ghardallou and Professor M. Fihri for organizing the satellite event on Non-Life Insurance, which will enrich the conference program and create a space for applied reflection and professional exchange.

We look forward to welcoming you to Rabat for this exciting scientific event and hope that you will enjoy both the academic richness of the program and the convivial spirit of our community.

Warm regards,

Pr. A. Zoglat
Chair of the Organizing Committee
JIMSA'2025

Contents

Introduction and Objectives

The Laboratory of Mathematics, Statistics, and Applications (LMSA) of the Department of Mathematics at the **Faculty of Sciences of Rabat (FSR)** has continuously contributed to the development of scientific research at Mohammed V University in Rabat, and consequently throughout Morocco, by training numerous PhD graduates each year and regularly organizing national and international conferences. Given the great success of previous conferences, the laboratory is organizing a new one titled:

International Days of Mathematics, Statistics and Applications

Journées Internationales de Mathématiques, Statistique et Applications

JIMSA'2025

This scientific event aims to be a platform for exchanging ideas and new results among prominent national and international mathematicians and the Moroccan mathematical community. It provides an opportunity for new PhD graduates, PhD students, and Master's students to present their ideas and findings, receive constructive criticism from experienced mathematicians, and make the most of these interactions.

JIMSA'2025 offers an ideal space for presenting new ideas and research results, as well as discussing current issues in various fields, including operator theory, function space theory, probability, statistics, and modeling.

The program will span three days (May 27 - 29, 2025) and will include: plenary sessions and parallel sessions.

These formats aim to foster exchanges between academics, researchers, and practitioners.

JIMSA'2025 will feature two complementary parallel sessions:

- Operator Theory, Function Spaces and Related Topics;
- Probability, Statistics and Modeling.

To ensure a dynamic and diverse scientific exchange, the Organizing Committee has planned three types of presentations:

- Plenary lectures of 40 minutes plus 5 minutes for discussion and questions.
- Oral presentations of 25 minutes plus 5 minutes for discussion and questions.
- Oral presentations of 12 minutes plus 3 minutes for discussion and questions.

As a satellite event, a (free) **doctoral school** will be held on **May 26-27, 2025**. This training session, particularly focusing on an introduction to non-life insurance, is aimed at PhD students and early-career researchers wishing to deepen their knowledge in this specific field.

The abstracts listed below have been reviewed by the scientific committee, which requested some authors to revise their submissions. The following versions are the latest received from the authors.

We look forward to welcoming you to **Rabat** and hope you enjoy your stay in this dynamic and evolving city.

For further inquiries, please contact professors:

- E. Benabdi – *Operator Theory, Function Spaces and Related Topics*: e.benabdi@um5r.ac.ma
- M. Fihri – *Probability, Statistics and Modeling*: m.fihri@um5r.ac.ma

Sessions & Topics

JIMSA'2025 features two sessions, each covering, but not limited to, a diverse range of research areas as follows:

Operator Theory, Function Spaces and Related Topics

- Operator Theory
- Preserver Problems
- Complex Analysis
- Function Spaces and Function Theory
- Operator on Analytic Function Spaces
- Operator-based Statistics
- Ulam type stability
- Topological linear spaces and related structures

Probability, Statistics and Modeling

- Probability and Stochastic Processes
- Modeling and Scientific Computing
- Artificial Intelligence and Machine Learning
- Multivariate Statistics
- Optimization and Operations Research
- Time Series Modeling and Extreme Value Statistics
- Risk Management and Quantitative Finance
- Mathematical Models for Insurance and Finance

Committees

The organization and success of ***JIMSA'2025*** rely on the commitment and expertise of several committees, bringing together renowned researchers and experts to ensure the scientific and logistical quality of the event.

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Program

Tuesday, May 27

14:00-15:00	**Welcome Participants – Registration**
15:00-16:00 **Belmahi Amphitheater**	***Opening Ceremony***
16:00-16:30	**Coffee Break and Official Photo**
16h30-17h15 **Chair: A. Zoglat**	***Opening Conference*** ***D. Ouazar*** Modeling paradigms: Conceptual, data-driven, and hybrid approaches – the future

Wednesday, May 28

- *Session* **Operator Theory, Function Spaces and Related Topics**

08:00-08:30	**Reception of participants**	
08:30-10:45	**Chair: M. Elhamdadi**	**Al Baytar Amphitheater**
08:30-09:15	**M. Mbekhta**	Polar factor: Approximation and preserver problems
09:15-10:00	**V. G. Bardakov**	Yang-Baxter equation and Rota-Baxter operators on groups, racks, and algebras
10:00-10:45	**K. Ezzinbi**	Linear dynamical systems and oscillatory dynamics for some evolution equations using Favard's theory in uniformly convex Banach spaces
10:45-11:15	**Coffee Break**	
11:15-12:45	**Chair: Z. Abdelali**	**Al Baytar Amphitheater**
11:15-11:45	**I. EL-Fassi**	Almost generalized multi-quadratic functions in Lipschitz spaces
11:45-12:15	**M. A. Ighachane**	Multiple-term improvements of Jensen's inequality for (p, h)-convex and (p, h)-log convex functions
12:15-12:45	**B. Seddoug**	On simple normal structure and best proximity points in reflexive Banach space
12:45-14:30	**Lunch Break**	

Wednesday, May 28

- **Session** **Operator Theory, Function Spaces and Related Topics**

14:30-16:45	Chair: V. Müller	Al Baytar Amphitheater
14:30-15:15	**L. Molnar**	Isomorphisms of positive cones in operator algebras under different types of geometric means
15:15-16:00	**E. Zerouali**	Propagation phenomena for subnormal matricial weighted shifts
16:00-16:45	**R. El Harti**	Crossed product Banach algebras associated with dynamical systems
16:45-17:15	**Coffee Break**	

17:15-19:15	Chair: K. Idrissi	Al Baytar Amphitheater
17:15-17:30	**C. Benzarouala**	A fixed point theorem in gauge spaces and applications to Ulam-stability of delay differential equations
17:30-17:45	**A. Ouannasser**	On anisotropic double phase problems
17:45-18:00	**H. Dkhissi**	Integral transform associated with a meromorphic Bargmann space
18:00-18:15	**S. Boudrai**	The slice hypermeromorphic Dirichlet space
18:15-18:30	**Y. El Khatiri**	Order isomorphisms on unbounded Self-adjoint Operators
18:30-19:45	**A. Ech-charyfy**	Infinite-dimensional flat extensions in operator moment problems
18:45-19:00	**R. Sersif**	Multiple solutions to the fractional (p,q)-Laplacian equations involving the critical exponents
19:00-19:15	**A. Retbi**	L-Dunford-Pettis property in Banach spaces

17:15-19:15	Chair: M. A. Ighachane	Daguin Room
	K. Azhoum	Notes on finite operator
	M. Barloub	The hessian equation in quaternionic space
	O. Abad	Insights on the closed generalized Drazin-R invertible operators and their applications to delay differential equations
	Z. Taki	About the numerical range of the basic elementary operator in semi-Hilbertian spaces
	A. El Gasmi	Degenerate Monge-Ampère equation on complex varieties in bounded domains of C^n
	A. Maarouf	Bargmann integral transform on the two-sided Bargmann-Fock space
	M. Masmodi	Multiple positive solutions for fractional elliptic problems with multi-critical Hardy-Littlewood-Sobolev exponents

17:15-19:15	Chair: M. Rossafi	Seminar Room, Department of Mathematics
17:15-17:30	**L. Bouali**	On Quaternionic Poly-Bargmann Spaces
17:30-17:45	**M. Morjane**	FPA-property
17:45-18:00	**S. Touaiher**	On K-g-Fusion Frames within Hilbert C*-Modules
18:00-18:15	**A. El Hyat**	Benedicks-Amrein-Berthier'S Uncertainty Principle for Quaternion Fourier Transform
18:15-18:30	**H. El Mouadine**	On the range of some elementary operators
18:30-18:45	**A. Bouhouch**	Analysis Calabi-Yau Geometry and Black Holes Physics
18:45-19:00	**S. Madani**	Putnam-Fuglede theorems and orthogonality of an elementary operator in Cp classes
19:00-19:15	**J. Boutarfass**	Ulam stability problem of certain functional equations

17:15-19:15	Chair: Y. Chatibi	Meeting Room, Department of Mathematics
	R. Bouydou	Well-posedness and convergence of a semi-discrete scheme for the conserving Allen-Cahn equation coupled to the unsteady Navier-Stokes equation
	H. Eddaoudi	Regularity of G(.)-superharmonic functions
	M. Boutrigue	Numerical range and numerical radius inequalities
	A. El Asri	The Lomonosov type theorems and the invariant subspace problem for non-archimedean Banach spaces
	R. Elhoua	Bargmann space associated to the two-sided slice regular functions on H^2
	K. Gouach	On the A-contractions operators
	I. Naainia	On the joint numerical radius of generalized spherical aluthge transforms of operators
	L. Yassine	Maps commuting with the lambda-Aluthge transform for the Lie Star Jordan product

Wednesday, May 28

- **Session** **Probability, Statistics and Modeling**

08:00-08:30	**Reception of participants**				
08:30-10:45	**Chair: A. Zoglat**	**Al Birouni Amphitheater**			
08:30-09:15	**Y. Ouknine**	Optimal Stopping under Model Uncertainty in a General Setting			
09:15-10:00	**B. Abou El Majd**	Du laboratoire à la réalité : valorisation de la recherche et innovation – L'exemple de MyMDO			
10:00-10:45	**N. Alaa**	On a Fractional Reaction–Diffusion System Applied to Image Restoration and Enhancement			
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11:15-12:45	**Chair: I. Bensaoud**	**Al Birouni Amphitheater**		**Chair: W. El Hannoun**	**Ibn Hayan Amphitheater**
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11:30-11:45	**F-Z. Ahssous**	Linear Quantile Mixed Models: From Theory to Implementation		**W. El Afari**	Advancing Brain Tumor Segmentation with Deep Learning: Insights from the BraTS Africa Dataset
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12:00-12:15	**S. Baqqass**	From ARIMA to Machine Learning: A Comparative Study on Daily High-Speed Rail Demand Forecasting		**I. Ennaqui**	Real-Time Planning of Customized Bus Routes Using Multi-Agent Deep Reinforcement Learning
12:15-12:30	**I. Benamara**	A Mathematical Modeling Approach to the Co-Dynamics of Covid-19 and Tuberculosis		**I. Essalih**	Systematic literature review of stability and hyperstability of some functional equations
12:30-12:45	**I. Chabba**	Forecasting Morocco's Electricity Imports: A Comparative Analysis between SARIMA and SVR		**K. El Qate**	Regularized Tensor Factorization for Hyperspectral Image Completion
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Wednesday, May 28

- *Session* **Probability, Statistics and Modeling**

14:30-16:45	**Chair: Z. El Hadri**	**Al Birouni Amphitheater**
14:30-15:15	**A. El Ghini**	Statistical Inference for the Generalized Autocorrelation Function of Non-Stationary Time Series under Weak White Noise: Theory and Applications
15:15-16:00	**S.-E. El Adlouni**	Régions climatiques de Köppen et informations a priori sur les extrêmes hydrologiques
16:00-16:45	**M. Hanafi**	A network approach to Joint Dimension Reduction of a set of data tables
16:45-17:15	**Coffee Break**	

17:15-19:15	**Chair: M. Ziani**	**Al Birouni Amphitheater**		**Chair: A. El Ghini**	**Ibn Hayan Amphitheater**
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17:30-17:45	**M. Idalfahim**	Optimization of Agricultural Production in the MENA Region Under Resource Constraints and Water Stress		**S. Ouhssaine**	A Novel Method for Estimating Structural Equation Models
17:45-18:00	**A. Imzourh**	Modeling Integer-Valued Time Series: Theory and Applications of INAR, INMA, and INARMA Processes		**M. Mimouni**	Physics-Informed Neural Networks for Parameter Estimation of SARS-CoV-2 Antigen-Antibody Interaction Dynamics
18:00-18:15	**S. Kahil**	A Hybrid Approach Combining K-Means Clustering and Machine Learning for Photovoltaic Power Prediction		**S. Mouchtabih**	Uniqueness of solutions to quadratic BSDEs with locally Lipschitz generator
18:15-18:30	**M. Khalil**	Proposal for an Intelligent Architecture for Resilient Smart Cities			
18:30-18:45	**I. Khaloufi**	Maximal Output Admissible Set for Linear Distributed Systems with an Application to Disturbance Rejection		**H. Laknaoui**	Proactive Threat Detection: Unsupervised Clustering Approaches for Anomaly Detection in Cybersecurity
18:45-19:00	**Y. Difaa**	Safety Distribution Analysis within Structured Epidemic Frameworks		**S. Kassimi**	Enhancing image denoising: A novel non-local anisotropic diffusion framework based on Caputo derivatives and Gaussian convolution for the Perona-Malik model
19:00-19:15	**A. Louakar**	Existence and optimal control of Hilfer fractional stochastic pantograph differential equations		**Z. Ghouli**	Energy Harvesting in a Time-Delayed Mathieu–Duffing MEMS Device
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Thursday, May 29

- **Session** **Probability, Statistics and Modeling**

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09:15-10:00	**A. Amar**	Weather Derivatives and Statistical Spatial Analysis: Factor and Markov Regime Switching Copulas			
10:00-10:45	**Y. Joulal**	Au-delà du hasard : modéliser le prix des options à l'ère des statistiques intelligentes			
10:45-11:15	**Coffee Break**				
11:15-12:45	**Chair: N. Ammor**	**Al Birouni Amphitheater**		**Chair: Z. Ghardallou**	**Ibn Hayan Amphitheater**
11:15-11:30	**H. Regragui**	Blind Deconvolution Using Game Theory and CNNs on Grayscale Image Datasets		**M. Slime**	Modeling Claim Costs and Premiums in Moroccan Auto Insurance with GLMs
11:30-11:45	**A. Sahli**	Overcoming convergence problems in PLS path modelling		**R. Taki**	Stationary distribution of a stochastic epidemic model with logistic growth
11:45-12:00	**M. Semlali**	The impact of white noise and Lévy jumps on the dynamics of an SIR epidemic model			
12:00-12:15	**F-Z. Semmane**	Blind deconvolution using bilateral total variation and nash equilibrium		**S. Tariq**	Statistical Aspects of Black Holes
12:15-12:30	**H. Zahraoui**	Regularized Generalized Canonical Correlation Analysis for multiblock data analysis		**A. Settar**	Log-volatility models in presence of zero returns : log-GARCH or SV ?
12:30-12:45	**A. Sghir**	Medical Image Segmentation with Dynamic Weighted Chan-Vese: Weight Prediction Using Gradient and CNN		**K. Zeroual**	Optimal control of cardiovascular diseases among HIV infected
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Thursday, May 29

- **Session** **Operator Theory, Function Spaces and Related Topics**

08:00-08:30	**Reception of participants**	
08:30-10:45	**Chair: M. Mbekhta**	**Al Baytar Amphitheater**
08:30-09:15	**V. Müller**	The essential numerical range in lp
09:15-10:00	**M. Barraa**	Joint numerical range of several quaternionic right linear bounded operators
10:00-10:45	**N. Boudi**	On the quantum phase operator
10:45-11:15	**Coffee Break**	
11:15-12:45	**Chair: E. Benabdi**	**Al Baytar Amphitheater**
11:15-11:45	**H. Ezzahraoui**	On the regularity of some bounded operators with closed ranges
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12:15-12:45	**S. Alaoui Chrifi**	Square roots of m-complex symmetric operators and permanence of spectral properties
12:45-14:30	**Lunch Break**	
14:30-16:45	**Chair: A. Zoglat**	**Al Baytar Amphitheater**
14:30-15:15	**L. Oubbi**	Generalized Nachbin weighted spaces, an overview
15:15-16:00	**A. Ghanmi**	An integral representation of the quaternionic Harmonic Bergman space
16:00-16:45	**M. Elhamdadi**	Idempotents in quandle rings
16:45-17:15	**Coffee Break**	
17:15-18:00	**Chair: E. Zerouali**	**Al Baytar Amphitheater**
17:15-18:00	**R. E. Curto**	The Local Operator Moment Problem on R
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Plenary Speakers

Operator Theory, Function Spaces and Related Topics

Yang-Baxter equation and Rota-Baxter operators on groups, racks, and algebras

Valeriy Georgievich Bardakov

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Abstract: The Yang-Baxter equation (YBE) is a fundamental equation arising in theoretical physics and has deep connections with mathematics especially braid groups and knot theory. It has been studied since 1960s-1970s, and there exist a lot of quite different versions of it. In 1992, V. G. Drinfeld highlighted the importance of the study of set-theoretical solutions to the YBE. Let X be a non-empty set and $S : X \times X \to X \times X$ be a bijection. The pair (X, S) is said to be a *set-theoretic solution of the Yang-Baxter equation* or simply a *solution of YBE*, if

$$S_1 S_2 S_1 = S_2 S_1 S_2,$$

where $S_1 = S \times \mathrm{id}$, $S_2 = \mathrm{id} \times S$ are bijections of $X \times X \times X$.

Let us recall the definition of Rota-Baxter operator on an algebra. Let A be an algebra over a field k. A linear operator R on A is called a *Rota-Baxter operator of weight* $\lambda \in \mathrm{k}$ if

$$R(x)R(y) = R(R(x)y + xR(y) + \lambda xy)$$

for all $x, y \in A$. An algebra endowed with a Rota-Baxter operator is called a *Rota-Baxter algebra*. The first appearance of Rota-Baxter operators (RB-operators) for commutative algebras can be traced back to Baxter's seminal paper [5] in 1960. Since then, the theory of Rota-Baxter operators has undergone extensive development by various authors in different fields of mathematics. It should be noted that Rota-Baxter operators play an important role due to their connection with a number of mathematical concepts, including the Yang-Baxter equation, Loday algebras, and double Poisson algebras. In 2021 L. Guo, H. Lang, Y. Sheng [6] defined a Rota-Baxter operator on groups. A *Rota-Baxter group* (RB-group) is a group G endowed with a map $B : G \to G$ satisfying the identity

$$B(g)B(h) = B(gB(g)hB(g)^{-1}),$$

where $g, h \in G$. In [6] was shown that if (G, B) is a Rota-Baxter Lie group, then the tangent map B at identity is a Rota-Baxter operator of weight 1 on the Lie algebra of the Lie group G.

The properties of RB-groups are actively studied in [2]. In [3], a connection between RB-groups, the Yang-Baxter equation and skew braces was found. Relative Rota-Baxter operators on an arbitrary Hopf algebra were defined in [4].
In [1] was introduced relative Rota-Baxter operators on racks. Let $(X, \cdot)$ and $(A, *)$ be racks and let $\Phi : A \to \mathrm{Conj}(\mathrm{Aut}(X))$ be a rack homomorphism. A *relative Rota-Baxter operator on* $(X, \cdot)$ *with respect to* (A, Φ) is a map $B : X \to A$ such that

$$B(x) * B(y) = B\left((x^{\Phi(B(y))}) \cdot y\right), \quad (x, y \in X)$$

where $\Phi(B(y)) := \Phi(B(y))$ is the image of $B(y)$ in $\mathrm{Aut}(X)$. The quadruple (X, A, Φ, B) is called a *relative Rota-Baxter rack*. If X and A are quandles, then the quadruple (X, A, Φ, B) is called a *relative Rota-Baxter quandle*.

In my talk I will give a survey of results which we have obtained during the last years and which are dedicated to the YBE, RB-operators, averaging operators, the rack, quandle, rack algebras and Hopf algebras.

Keywords: Yang-Baxter Equation, Rota-Baxter operator, quandle.
2020 Mathematics Subject Classification: 20E07, 20F36, 57K12.

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Joint numerical range of several quaternionic right linear bounded operators

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Abstract: In [1, 2, 3], some properties of the numerical range of a quaternionic right linear operator are studied. The purpose of this note is to study analogous results concerning the joint numerical range of an n-tuple of quaternionic right linear operators.

Keywords: Numerical range, quaternionic linear operator.
2020 Mathematics Subject Classification: 47A12, 47S10.

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On the quantum phase operator

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Abstract: An interesting mathematical problem, initially related to the study of the electromagnetic field, is how to construct a self-adjoint operator Θ associated to the time phase variable θ for the quantum harmonic oscillator (QHO). Many authors worked on this problem and we may distinguish 3 main directions:

1. There is no phase operator Θ and authors focus on functions of the phase, like $\cos\Theta$ and $\sin\Theta$.
2. The Hilbert space $\mathcal{H}_{QHO}$ associated to the QHO is extended and Θ can be constructed.
3. The finite dimensional approach.

In this talk, I shall explain the construction of the operators $\cos\Theta$, $\sin\Theta$ and $e^{-i\Theta}$ in the Susskind-Glogower formalism of the quantum phase, the Garrison-Wong construction of Θ in the Hardy space $H^2(\mathbb{D})$, Newton's construction of Θ in an extension of $\mathcal{H}_{QHO}$ and the Barnett-Pegg finite dimensional approach. Then I shall examine some algebraic relations that should be satisfied by Θ and by the exponential phase operator $e^{-i\Theta}$, and show that these relations lead naturally to a self-adjoint Θ in a natural extension of $\mathcal{H}_{QHO}$. Finally, I shall discuss the new commutation relations (for the extensions of the ladder operators) and compare with other constructions.

Keywords: Phase operator, exponential phase operator, number operator, self-adjoint operators, unitary operators, representation of commutation relations.
2020 Mathematics Subject Classification: 47B93, 47B02, 47B35.

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The local operator moment problem on $\mathbb{R}$

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Abstract: We study the connections between operator moment sequences $\mathcal{T} = (T_n)_{n\in\mathbb{Z}_+}$ of self-adjoint operators on a complex Hilbert space $\mathcal{H}$ and the local moment sequences $\langle \mathcal{T}x, x\rangle = (\langle T_n x, x\rangle)_{n\in\mathbb{Z}_+}$ for arbitrary $x \in \mathcal{H}$. We provide necessary and sufficient conditions for solving the operator moment problem on $\mathbb{R}$, and we show that these criteria are automatically valid on compact subsets of $\mathbb{R}$. Applications of the compact case are used to study subnormal operator weighted shifts.

A Stampfli-type propagation theorem for subnormal operator weighted shifts is also established. In addition, we discuss the validity of Tchakaloff's Theorem for operator moment sequences with compact support. In the case of a recursively generated sequence of self-adjoint operators, necessary and sufficient conditions for an affirmative answer to the operator recursive moment problem are provided, and the support of the associated representing operator-valued measure is described.

The talk is based on joint work with Abderrazzak Ech-charyfy (Mohammed V University in Rabat), Hamza El Azhar (Chouaib Doukkali University) and El Hassan Zerouali (Mohammed V University in Rabat).

Keywords: Operator-valued measures, operator moment problem, local operator moment problem, recursive sequences, subnormal operator weighted shifts.
2020 Mathematics Subject Classification: 47B15, 46G10, 44A60, 47A20.

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Crossed product Banach algebras associated with dynamical systems

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Abstract: From a locally compact group, a concrete Banach algebra, and an action of this group on this algebra, we construct an equally concrete algebra called Crossed Product Banach algebra asspcieted with a dynamical system. The study of this class of algebras plays a very important role in abstract harmonic analysis and representation theory. Moreover, WE wonder when this class is Hermitian or amenable.

Keywords: C*-algebras, locally compact groups, representations on Hilbert spaces.
2020 Mathematics Subject Classification: Primary 46L05, 46L07; Secondary 43A07, 43A65.

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Idempotents in quandle rings

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Abstract: Quandles are algebraic structures motivated by Reidemeister moves in knot theory. We will review the basics of quandles and then consider quandle rings. We will discuss idempotents in quandle rings, show how they can be used to construct invariants of knots. Some conjectures will be stated.

Keywords: Knots, quandles, idempotents in quandle rings.
2020 Mathematics Subject Classification: 20N02, 17D99.

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Linear dynamical systems and oscillatory dynamics for some evolution equations using Favard's theory in uniformly convex Banach spaces

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Abstract: In this work, we use an approach due to Favard [1] to study the existence of weakly almost periodic and almost automorphic solutions for some evolution equation whose linear part generates a C_0-group satisfying the Favard condition in uniformly convex Banach spaces. When this C_0-group is bounded, which is a condition stronger than Favard's condition, we prove the equivalence between almost automorphy and weak almost automorphy of solutions.

Keywords: Evolution equation, C_0-group, Favard condition.
2020 Mathematics Subject Classification: 34G10, 34K14, 35B15.

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An integral representation of the quaternionic Harmonic Bergman space

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Abstract: We study the quaternionic analog of the harmonic Bergman space within the framework of slice harmonic functions. We provide in particular additive decomposition theorems which will be employed to derive the integral representation of Bargmann type of the considered space.

The talk is based on joint work with Lamya Bouali (Mohammed V University in Rabat) and Youssef Laaouisset (Ibn Tofail University in Kenitra).

Keywords: Slice regular functions; slice harmonic Bergman space; Bargmann transform.
2020 Mathematics Subject Classification: Primary 30G35; Secondary 44A20, 30F15.

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Polar factor: Approximation and preserver problems

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Abstract: This talk is divided into two main parts. The first part will be devoted to the approximation of the polar factor and the second will deal with the characterization of maps that preserve the polar factor.

The talk is based on joint work with Abdellatif Bourhim (Syracuse University, USA).

Keywords: Polar decomposition, polar factor, preserver problems.
2020 Mathematics Subject Classification: 47A58, 41A35, 46A32.

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Isomorphisms of positive cones in operator algebras under different types of geometric means

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Abstract: This talk is divided into two main parts. The first part will be devoted to the approximation of the polar factor and the second will deal with the characterization of maps that preserve the polar factor.

Keywords: Geometric means, positive definite cone, C^*-algebra, preservers.
2020 Mathematics Subject Classification: 47A64; 47B49; 47L07; 46L40.

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The essential numerical range in ℓ_p

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Abstract: By a classical result of Hausdorff-Toeplitz, the numerical range of Hilbert space operators is always a convex subset of the complex plane. There are many variants of the notion of numerical range in Hilbert spaces, which are convex. However, the convexity is closely related with Hilbert spaces. The numerical range of Banach space operators is practically never convex.

Keywords: Essential numerical range, Banach spaces.
2020 Mathematics Subject Classification: 47A12.

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Generalized Nachbin weighted spaces, an overview

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Abstract: Weighted spaces of continuous functions were introduced by L. Nachbin in 1964, in connection with approximation theory [3]. These are linear spaces of continuous functions defined by some growth conditions, equipped with a family of weighted uniform norms. Subsequently similar spaces of holomorphic or harmonic functions were studied intensively. Actually, many of the natural spaces encountered in analysis fall in this category of spaces.
If A is a linear space of continuous functions from a Hausdorff completely regular space X into a Hausdorff locally convex space $(E, \mathbb{P})$, and V is a family of non-negative upper semi-continuous mappings on X, then the associated weighted space is :

$$AV(X,E) = \{f \in C(X,E); vf : x \mapsto v(x)f(x) \text{ is bounded}\}.$$

Endowed with the topology $\tau_{V,\mathbb{P}}$ given by the semi-norms:

$$P_v(f) := \sup\{v(x)P(f(x)), x \in X\}, \quad f \in AV(X,E)$$

with $P \in \mathbb{P}$ and $v \in V$.
In this talk, we will consider, instead of scalar-valued weights, a family V of weights with values in the continuous operators on E. We then show the pertinence of this consideration and obtain some results in this general context concerning multiplication operators on the so-obtained locally convex space $CV(X,E)$ [4, 5], as well as a Banach-Stone type theorem on $HV(X,E)$ [6], in the particular case where X in an open subset of $\mathbb{C}^n$ and E is a Banach space and V consists of a single weight v.

Keywords: Generalized weighted spaces, multiplication operator, Banach-Stone theorem
2020 Mathematics Subject Classification: 47B38, 46E10, 47A56, 46E15, 46A03.

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Propagation phenomena for subnormal matricial weighted shifts

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Abstract: The propagation Stampfli's theorem states that for a subnormal weighted shift, all weights (excluding the first one) are equal, when any two consequetif weight are. We discuss in this talk the case of matricial subnormal weighted shifts and provide an analog of Stampfli's Theorem on flat propagation for the more general case 2-hyponormal matricial weighted shifts.

The talk is based on joint work with Raúl E. Curto (The University of Iowa, USA), Abderrazzak Ech-charyfy (Mohammed V University in Rabat), Hamza El Azhar (Chouaib Doukkali University) and Kaissar Idrissi (Mohammed V University in Rabat).

Keywords: Matrix-valued measures, Matrix moment problem, subnormal matricial weighted shifts, propagation phernomena.
2020 Mathematics Subject Classification: 47B15,46G10,44A60,47A20.

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Probability, Statistics and Modeling

Du laboratoire à la réalité : valorisation de la recherche et innovation L'exemple de MyMDO

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Abstract: Research valorization is a strategic challenge in today's world enabling scientific discoveries to become real-world innovations with technological, economic, and societal impact.

In this talk, I will provide both a conceptual and practical perspective on the valorization process, using concrete methodologies and real-life examples. A key focus will be the MyMDO project, a cloud-based platform for multidisciplinary shape optimization, developed within a tech transfer framework.

The goal is to illustrate how a research project can be structured, valorized, and transitioned into an innovation-driven initiative – potentially leading to a startup or an impactful technology. This session is tailored for PhD students, researchers, and project holders who aim to bring their science closer to implementation and value creation.

A bridge between science and action, between academic knowledge and transformative innovation.

On a Fractional Reaction–Diffusion System Applied to Image Restoration and Enhancement

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Abstract: In this talk, we introduce a novel reaction-diffusion system governed by the fractional Laplacian for digital image restoration and contrast enhancement.

Our model employs a Sobolev-type fractional semi-norm, which offers several significant advantages: it preserves high-frequency edge features while simultaneously enhancing low-frequency texture details in smoother regions. The fractional Laplace operator in our formulation depends solely on pixel location and gray-level intensity.

By incorporating a reaction term into the model, we further improve contrast enhancement and overall image quality.

Based on these features, we demonstrate that the proposed model is well-posed. As a first result, we establish the existence of a weak solution under the assumption that the reaction terms are bounded. Then, using an approximation approach, we prove the existence of a weak, non-negative SOLA (Solution Obtained as the Limit of Approximations) for the model.

Finally, numerical experiments show that our model outperforms existing approaches in the literature, particularly in image enhancement and denoising tasks.

Keywords: Laplace Fractional Operator, Reaction Diffusion, Image Restoration, Image Enhancement.
2020 Mathematics Subject Classification: Primary 35K55, 35A02, 35R11, 68U10, 65J15.

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Market power: A powerful motive for mergers in extractive industries

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Abstract: We examine firms' incentives to acquire rivals in an effort to monopolize an exhaustible resource sector, and the equilibrium industry structure that emerges, when the acquisition price is endogenous. Firms compete in quantities: each entity chooses its extraction policy, i.e. a Markovian strategy that allows extraction rate to depend on the vector of stocks. When firms' stocks are sufficiently small, in contrast to the static Cournot case, monopolization is a profitable strategy. The firm with the largest stock is the least likely to monopolize the industry. The lower the demand elasticity, the less likely that either extreme case, i.e. monopoly or the unmerged equilibrium, occurs. We also show that a carbon tax may deter monopolization and thereby speed up the extraction of fossil fuel.

Régions climatiques de Köppen et informations a priori sur les extrêmes hydrologiques

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Abstract: La distribution généralisée des valeurs extrêmes (GEV) couvre plusieurs modèles pour la modélisation des événements extrêmes. L'approche du maximum de vraisemblance généralisée (GML) a apporté des solutions aux problèmes de convergence des approches classiques vers des solutions non-acceptables pour l'estimation des paramètres. L'approche GML considère la même distribution a priori pour restreindre l'intervalle des solutions du paramètre de forme de la GEV. L'objectif de cette étude est de caractériser la queue de la distribution des crues extrêmes en fonction des régions climatiques de Köppen sur la base de plus de 4000 stations hydrométriques. Des distributions a priori ont été considérées pour proposer une extension (EGML) de l'approche GML et tenir compte des réponses hydrologiques pour quatre principales régions climatiques des cinq continents. Nous présenterons les différentes étapes d'apprentissage pour déterminer les régions homogènes et les distributions a priori associées. Les résultats montrent l'importance d'un choix adéquat de la région climatique pour une estimation efficace des quantiles extrêmes.

Statistical Inference for the Generalized Autocorrelation Function of Non-Stationary Time Series under Weak White Noise: Theory and Applications

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Abstract: The spectral analysis of integrated time series is well known to be challenging due to the failure of classical Fourier methods in such cases. To address this, several approaches have been proposed for the spectral analysis of non-stationary processes. Hatanaka and Suzuki (1967) developed a spectral theory for non-stationary processes by focusing on finite subsets of the sequence with finite second moments. The resulting pseudo-spectrum is time-varying and closely related to Priestleys (1981) concept of evolutionary spectra. Building on these foundations, Baragona and Battaglia (1995) defined the inverse autocovariance function of a non-stationary time series as the Fourier coefficient of its inverse pseudo-spectrum.

In this paper, we investigate the consistency and asymptotic normality of sample generalized autocorrelations in the context of non-stationary time series driven by weak white noise. Statistical inference in this framework is particularly challenging due to the dependence structure of the generating noise. Additionally, we characterize the inverse process generated by ARIMA models, drawing on several intermediate results, particularly those established in El Ghini (2010) for stationary settings.

The main focus of the paper is on establishing the consistency and asymptotic properties of our estimators. These theoretical results are supported by simulations based on a variety of synthetic datasets.

Keywords: Non-stationary time series, ARCH models, consistency, asymptotic normality, α-mixing dependence, generalized (inverse) autocorrelations.

A network approach to Joint Dimension Reduction of a set of data tables

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Abstract: To deal with the dimension reduction task simultaneously of different classes of multi-block data, we propose to model theses classes in the form of graphs, introducing the notion of networks between data tables. Several examples are presented to illustrate this new notion and its generic nature. A generalization of the known Eckart-Young problem from a single table to a network of tables is formulated leading to an exploratory method for analyzing a network of tables. Main steps of an ALS type algorithm for solving this problem are described. An illustration based on real data is presented.

Au-delà du hasard : modéliser le prix des options à lère des statistiques intelligentes

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Abstract: La modélisation du prix des options constitue lun des champs les plus étudiés de la finance quantitative. Si les modèles classiques, tels que celui de Black-Scholes-Merton, ont permis une avancée majeure en matière de valorisation des instruments dérivés, ils reposent sur un certain nombre dhypothèses statistiques notamment la normalité des rendements, la constance de la volatilité et labsence deffets de mémoire qui savèrent limitantes dans les conditions de marché réelles.
Cette conférence sintéresse à lévolution des approches statistiques appliquées au pricing des options, en mettant en lumière lapport de méthodes avancées telles que lanalyse de séries temporelles non stationnaires, les modèles à volatilité stochastique, les processus de Lévy, ou encore les techniques issues de lapprentissage statistique. Lobjectif est de montrer comment ces outils permettent de mieux capturer la complexité des dynamiques de marché et daffiner la précision des modèles de valorisation.
En articulant rigueur mathématique et ancrage empirique, cette intervention vise à ouvrir une réflexion sur la manière dont les statistiques, au-delà de leur rôle doutil, deviennent un **cadre structurant de la pensée financière contemporaine**, notamment dans un contexte de marchés instables et de données massives.

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- Ouvrage francophone alliant rigueur mathématique et applications concrètes : diffusion stochastique, estimation statistique, pricing.

Modeling Paradigms: Conceptual, Data-Driven, and Hybrid Approaches The Future

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Abstract: Modeling lies at the heart of scientific discovery and engineering innovation. Traditionally, conceptual models – based on physical laws, mathematical formulations, and expert knowledge – have provided clarity, structure, and predictability in understanding complex systems. In parallel, the rise of data-driven models, powered by advances in machine learning and artificial intelligence, offers new capabilities for pattern recognition, real-time forecasting, and adaptive control, especially in systems where first-principles knowledge is incomplete or evolving. Bridging these two worlds, hybrid modeling approaches are rapidly gaining traction, combining theoretical rigor with empirical flexibility to tackle challenges across engineering domains and scientific inquiry.

This conference will explore the evolving landscape of modeling paradigms with a focus on engineering and scientific applications. Topics include the integration of physics-informed machine learning, uncertainty quantification, model validation, and the development of models that are both interpretable and deployable. By bringing together researchers and practitioners from diverse fields, we aim to advance a unified vision for modeling that leverages both domain knowledge and data, enabling robust decision-making, system optimization, and innovation in an increasingly complex and data-rich world.

Optimal Stopping under Model Uncertainty in a General Setting

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Abstract: We consider the optimal stopping time problem under model uncertainty $R(v) = \operatorname{ess\,sup}_{\mathbb{P}\in\mathcal{P}} \operatorname{ess\,sup} E^{\mathbb{P}}[Y(\tau) \mid \mathcal{F}_v]$, for every stopping time v, set in the framework of families of random $\tau \in \mathcal{S}_v$ variables indexed by stopping times. This setting is more general than the classical setup of stochastic processes, and particularly allows for general payoff processes that are not necessarily right-continuous. Under weaker integrability, and regularity assumptions on the reward family $Y = (Y(v), v \in S)$, we show the existence of an optimal stopping time. We then proceed to find sufficient conditions for the existence of an optimal model. For this purpose, we present a universal optional decomposition for the generalized Snell envelope family associated with Y. This decomposition is then employed to prove the existence of an optimal probability model and study its properties[1].

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Talk Abstracts

Operator Theory, Function Spaces and Related Topics

Insights on the closed generalized Drazin-$\mathcal{R}$ invertible operators and their applications to delay differential equations

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Abstract: In this talk, we aspire to introduce and characterize generalized Drazin invertible operators relative to a regularity $\mathcal{R}$ to the realm of closed operators. Also, we state new applications of this notion in relation with delay differential equations and also with second-order partial differential equations.

Keywords: Generalized Drazin inverse, Regularity, closed linear operators, delay differential equations.
2020 Mathematics Subject Classification: Primary 47A10, 34K30, 47D03.

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Square roots of m-complex symmetric operators and permanence of spectral properties

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Abstract: A bounded linear operator T on a separable Hilbert space H is called the square root of an m-complex symmetric operator if there exists a conjugation C such that $\Delta_m(T^2) = 0$, where $\Delta_m(T) = \sum_{j=0}^{m}(-1)^{m-j}\binom{m}{j}T^{*j}CT^{m-j}C$. In this presentation, we explore local spectral relations between an arbitrary operator T and its adjoint T^*, when T is a square root of an m-complex symmetric operator. In particular, we study the transmission from T^* to T of the single-valued extension property (SVEP), property (β), and decomposability. We also establish additional spectral properties and construct commuting nilpotent perturbations for this class of operators.

Keywords: square root of m-complex symmetric operator, spectral and local spectral properties, nilpotent perturbations.
2020 Mathematics Subject Classification: Primary 47A05, 47A11, 47B25.

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On the △-Finite Operators And Their Applications

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Abstract: Let $\mathcal{H}$ be a separable infinite dimensional complex Hilbert space, and $\mathcal{L}(\mathcal{H})$ denotes the algebra of all bounded linear operators on $\mathcal{H}$. A new class of operators is introduced, called $\Delta\mathcal{F}(\mathcal{H})$-finite operators and denoted by $\Delta\mathcal{F}(\mathcal{H})$, defined by:

$$\Delta\mathcal{F}(\mathcal{H}) = \{A \in \mathcal{L}(\mathcal{H}) : \|AXA - X - I\| \geq 1, (\forall X \in \mathcal{L}(\mathcal{H}))\}.$$

Basic properties of this class are given and some examples are presented. We prove that a spectraloid operator or class $\mathcal{Y}$ under certain scalar perturbation is Δ-finite operator. We also give an engineering application.

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THE HESSIAN EQUATION IN QUATERNIONIC SPACE

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Abstract: In this paper, we introduce m-subharmonic functions in quaternionic space $\mathbb{H}^n$, we define the quaternionic Hessian operator and solve the homogeneous Dirichlet problem for the quaternionic Hessian equation on the unit ball with continuous boundary data.

Keywords: Potential theory in quaternionic space, m-subharmonic function, quaternionic Hessian equation, Dirichlet problem.
2020 Mathematics Subject Classification: 32U15, 35J60.

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A fixed point theorem in gauge spaces and applications to Ulam-stability of delay differential equations

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Abstract: During this talk, we prove a new alternative fixed point theorem in generalized gauge (or generalized uniformizable) spaces. This is a generalization of a famous result of Diaz-Margolis. Next, using this theorem, we show the stability of the following delay differential equation

$$y^{'}(t) = F(t, y(t), y(g(t))), \quad t \in I \subset \mathbb{R},$$

where the unknown mapping y takes its values in a locally convex space. Examples are given to support our results. This talk is based on the article [1].

Keywords: Ulam-Hyers-stability, Ulam-Hyers-Rassias-stability, Delay differential equation, Direct method, Fixed point theorem, Locally Convex space, Riemann integral, Generalized Gauge spaces, Generalized pseudo-metric, Yanyan-continuous mapping.
Joint work with Lahbib Oubbi
2020 Mathematics Subject Classification: 39B05, 39B82, 54E70, 47H10

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On quaternionic poly-Bargmann spaces

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Abstract: We construct the n-th true S-poly-Bargmann space as the range of the iterated sliced creation operator, which proved to closely connected to the spectral analysis of a sliced magnetic Laplacian.

Keywords: S-polyregular functions; S-poly-Bargmann space; Sliced creation operator; Sliced magnetic Laplacian.
2020 Mathematics Subject Classification: Primary 32A36 · 30G35; Secondary 32A20, 44A20.

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The slice hypermeromorphic Dirichlet space

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Abstract: In the present work, we introduce and investigate the slice hypermeromorphic Dirichlet space by studying its analytic properties. Moreover, we give the closed expression of its reproducing kernel function.

Keywords: Slice regular functions; Dirichlet space; Reproducing kernel function; Hypergeometric functions.
2020 Mathematics Subject Classification: Primary 32A36, 30G35, Secondary 32A20, 44A20 .

On the Range-Kernel Orthogonality of Elementary Operators

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Abstract: Let $L(H)$ denote the algebra of operators on a complex infinite dimensional Hilbert space H and let $(\mathcal{J}, \|\cdot\|_{\mathcal{J}})$ denote a norm ideal in $L(H)$. Given $A, B, C, D \in L(H)$, we define the generalized derivation $\delta_{A,B}$ and the elementary operator Φ by $\delta_{A,B}(X) = AX - XB$ and $\Phi(X) = AXD - CXB$. We give some pairs (A, B) of operators A and B such that the range $R(\delta_{A,B} \mid \mathcal{J})$ is orthogonal to the kernel $\ker(\delta_{A,B} \mid \mathcal{J})$ of the restriction $\delta_{A,B} \mid \mathcal{J}$ with respect to unitarily invariant norm $\|.\|_{\mathcal{J}}$. We establish the orthogonality of the range and the kernel of a derivation δ_A induced by a rationally cyclic subnormal operator A in the usual operator norm. Furthermore, We study the range-kernel orthogonality of the elementary operators Φ.

Keywords: Fuglede-Putnam property, Orthogonality, Elementary operator, norm ideal, Log-hyponormal, w-hyponormal.
2020 Mathematics Subject Classification: Primary 47A30, 47A63, 47B15, 47B20, 47B47, 47B10.

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Analysis Calabi-Yau Geometry and Black Holes Physics

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Abstract: Combining complete intersections in prejective spaces or toric geometry techniques and $\mathcal{N} = 2$ supergravity formalisms, we study 5D black branes in the M-theory compactification on a two, three and four parameter Calabi-Yau threefold. First, we investigate 5D BPS and non-BPS black holes that are derived by wrapping M2-branes on non-holomorphic 2-cycles in such a Calabi-Yau manifold. Concretely, we provide the allowed electric charge regions of BPS and non-BPS black hole states that are obtained by surrounding M2-branes over appropriate 2-cycles. Then, we approach the black hole thermodynamic behavior by computing the entropy and the temperature. By evaluating the recombination factor, we examine the stability of such non-BPS black holes. Precisely, we find stable and unstable solutions depending on the allowed electric charge regions. After that, we study 5D black strings by wrapping M5-branes on non-holomorphic dual 4-cycles in the proposed complete intersections and toric Calabi-Yau manifolds by focusing on the stability behaviors. In the allowed regions of the moduli space of the non-BPS stringy solutions, we find stable and unstable states depending on the magnetic charge values. Finally, we propose a general idea to integrate machine learning to study the stability behaviors of all non BPS black objects of M theory on a Complet Intersections Calabi-Yau three-folds models in terms of Kahler parameters determined by hodge numbers $h^{1,1}$.

Keywords: 5D $N = 2$ supergravity formalism, Black holes, Black strings, Calabi-Yau manifolds, Stability Behaviors.

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Ulam stability problem of certain functional equations

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Abstract: It is well known that the stability problem of functional equations was motivated by a question posed by Ulam [4] in 1940, with a positive answer provided by Hyers [3] the following year. Ulam stability theory has found interesting applications in various fields, such as probability theory (see [2]), stochastic analysis [5], financial and actuarial mathematics, as well as in psychology and sociology. The main purpose of this research is to investigate the hyperstability of generalized derivation in quasi-Banach algebras. More precisely, using the direct method (commonly referred to as Hyers method), we prove that, under natural conditions, every approximate generalized derivation in a quasi-Banach algebra is in fact an exact generalized derivation [1].

Keywords: stability, functional equations, derivations, quasi-Banach algebras
2020 Mathematics Subject Classification: 39B82, 39B62, 46J10, 16W25, 16N60.

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Numerical Range and Numerical Radius Inequalities

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Abstract: In this presentation, we explore inequalities involving the numerical range and numerical radius of bounded linear operators, with a focus on matrix cases. The numerical range is a fundamental tool in operator theory due to its convexity and its role in localizing the spectrum of an operator. Starting from classical results, we derive improved bounds for the numerical radius and investigate their consequences for spectral estimates and norm control. The Frobenius companion matrix serves as a motivating example, highlighting how algebraic structures influence operator behavior. Through this matrix, we show how numerical range techniques can provide insight into the location of polynomial roots and the geometry of associated operators.

Keywords: Numerical range, numerical radius, operator inequalities, spectrum localization, convexity, Frobenius companion matrix, operator norm, spectral theory, polynomial roots.
2020 Mathematics Subject Classification: Primary 47A12, 15A60, 47A05; Secondary 30C15, 65F15.

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Well-posedness and convergence of a semi-discrete scheme for the conserving Allen-Cahn equation coupled to the unsteady Navier-Stokes equation

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Abstract: In the broad spectrum of approaches to model multi-fluid (multi-phase) flow and the capture of its interfacial behaviour, the phase-field approach is quite attractive as it is a physically motivated model based on the competition between the different species/phases. We refer the readers to [2] for a review of some of the most common approaches (both theoretically and numerically). This work is part of an effort to produce a new numerical algorithm for the approximation of the complex behavior of a binary mixture of fluids. More precisely, we are interested in the use of the conserving Allen-Cahn model [3, 4] for the description of the dynamic of the mixture of two non miscible fluids coupled to the unsteady Navier-Stokes equations describing fluids displacements. This leads to what is known as an unsteady *Navier-Stokes-Allen-Cahn* (NS-AC) model [1]. In appropriate functional spaces, we proved the existence and uniqueness of the solution to a nonlinear system of equations arising from the implicit time discretization of the coupled unstable Allen-Cahn and Navier-Stokes equations. The aforementioned outcome stems from the examination of the convergence of an initial stabilized fixed point algorithm, which also yields a maximal principle.

Keywords: Navier-Stokes, Allen-Cahn, implicit time scheme, fully implicit strategy, well-posedness, convergence.
2020 Mathematics Subject Classification: 65M12, 65M70, 65P99, 65Z05, 76T99.

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Integral transform associated with a meromorphic Bargmann space

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Abstract: We consider two classes of poly-meromorphic Bargmann spaces generalizing the classical Segal-Bargmann spaces.

Keywords: Modified Bargmann Fock space, Reproducing kernel.
2020 Mathematics Subject Classification:

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Infinite-Dimensional Flat Extensions in Operator Moment Problems

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Abstract: In this work, we extend the notion of flatness in operator moment problems to the infinite-dimensional setting. We introduce a general definition of flatness for self-adjoint operators and provide several characterizations that generalize the finite-dimensional case. Furthermore, we establish a partial result showing that the flatness condition ensures the existence of operator-valued representing measures, which are not necessarily finitely atomic.

Keywords: Operator moment problem, representing measures, flat extension, recursive sequences.
2020 Mathematics Subject Classification: Primary 44A60; Secondary 47A57, 46G10.

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The regularity of $G(\cdot)$-superharmonic function

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Abstract: In this talk, we study local integrability properties of superharmonic functions related to partial differential equations with Musielak-Orlicz growth conditions in Lebesgue and Musielak-Orlicz spaces.

Keywords: $G(\cdot)$-superharmonic, Local integrability, Musielak-Orlicz growth, Generalized Φ-function.

2020 Mathematics Subject Classification: 35B65. 35J62. 46E30

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ALMOST GENERALIZED MULTI-QUADRATIC FUNCTIONS IN LIPSCHITZ SPACES

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Abstract: The notion of stability of functional equations was posed by Ulam [3]. In 1941, Hyers [1] gave a first affirmative partial answer to the question of Ulam for Banach spaces. This type of stability has been established and developed by an increasing number of mathematicians in various spaces. In Lipschitz spaces, the notion of stability was introduced by Tabor [2]. In this work, we describe the general solution of a generalized multi-quadratic functional equation, and under some natural conditions, we discuss the stability results for the same functional equation. We conclude this work by presenting some important consequences.

Keywords: Stability, Functional equations, Symmetric left invariant mean, Lipschitz spaces.
2020 Mathematics Subject Classification: Primary 39B82, 39B52, 39B72.

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The Lomonosov type theorems and the invariant subspace problem for non-archimedean Banach spaces

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Abstract: In this talk, we investigate the existence of invariant and hyperinvariant subspaces for bounded operators on non-archimedean Banach spaces $\mathbf{E} = (\mathbf{E}, \|\cdot\|)$ over a non-trivially valued field $\mathbb{K}$. Our main focus is on operators that commute with compact operators. Furthermore, we examine the validity of the classical Lomonosov Invariant Subspace Theorem in the non-archimedean setting, highlighting key differences from the classical one. This talk is based on a joint work with A. Kubzdela and M. Babahmed [2].

Keywords: Invariant subspace; Hyperinvariant subspace; Compact operator; Non-archimedean Banach space.
2020 Mathematics Subject Classification: 46S10.

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Equation de Monge-Ampère dégénérée sur une variété complexe dans un domaine borné de $\mathbb{C}^n$

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Abstract: We study the Dirichlet problem in Cegrell classes for non-negative Radon measure μ that can eventually charge pluripolar subsets of V. We prove that if V is a locally irreducible hyperconvex variety in a bounded domain of $\mathbb{C}^n$ and μ is a non-negative measure in V such that $\mu \leq (dd^c w)^k$ with $w \in \mathcal{E}(V)$, then there exists a function $u \in \mathcal{E}(V)$ such that $(dd^c u)^k = \mu$.

Keywords: Plurisubharmonic functions, Monge-Ampère operator, Dirichlet problem, Complex varieties.
2020 Mathematics Subject Classification: 32U15, 32B15, 32W20.

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Benedicks-Amrein-Berthier'S Uncertainty Principle for Quaternion Fourier Transform

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Abstract: Uncertainty principle (UP) plays an important role in signal processing and physics, it states that a non-zero function and its Fourier transform cannot both have a finite measure support. The objective of this presentation is to discuss Benedicks, Amrein and Berthier's UP for the two-sided quaternion Fourier transform.

Keywords: Uncertainty Principle, Benedicks-Amrein-Berthier type theorem, Quaternion Fourier Transform.
2020 Mathematics Subject Classification: 30G30, 42B10, 43A32.

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Order isomorphisms on unbounded self-adjoint operators

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Abstract: We provide a comprehensive description of all order isomorphisms between several types of unbounded self-adjoint operator sets. Namely, sets of all positive operators, sets of all positive boundedly invertible operators, and those of all self-adjoint operators. To achieve a complete description of these isomorphisms, we will discuss, the possibility for two sets of the previously mentioned types, defined either on a same Hilbert space or on two different Hilbert spaces, to be order isomorphic.

Keywords: Self-adjoint, Unbounded operator, Order isomorphism, Preservers.
2020 Mathematics Subject Classification: Primary 15A86 ; Secondary 46B40, 47B25.

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On the range of some elementary operators

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Abstract: Let $L(H)$ denotes the algebra of all bounded linear operators on a complex infinite dimensional Hilbert space H. For $A, B \in L(H)$, the generalized derivation $\delta_{A,B}$ and the multiplication operator $M_{A,B}$ are defined on $L(H)$ by $\delta_{A,B}(X) = AX - XB$ and $M_{A,B}(X) = AXB$. We shall give a characterization of bounded operators A and B such that the range of $M_{A,B}$ is closed. We present some sufficient conditions for $\delta_{A,B}$ to have closed range. Some related results are also given.

Keywords: generalized derivation; elementary operator; generalized inverse; Kato spectrum.
2020 Mathematics Subject Classification: 47A30, 47A16, 47B07, 47B20, 47B47.

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Bargmann space associated to the two-sided slice regular functions on $\mathbb{H}^2$

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Abstract: Participants are required to submit an abstract in English.
We introduce and study the algebraic and analytic strucure off the so-colled quaternionic left-right slice regular functions on $\mathbb{H}^2$. The Bargmann space is also studied.

Keywords: slice regular functions, two-sided Bargmann space, middle hilbertian structure,...

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On the regularity of some bounded operators with closed ranges

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Abstract: The aim of this presentation is to expose two classes that are close to class of regular operators: The classes of quasi regular operators and that of power relatively regular. These two classes are larger than the classical class of regular operators. In particular, they contain nonzero operators that are not necessarily regular. We will show that they have properties similar to those of regular operators. As applications of this study, we give new characterizations of regular operators and that of power partial isometries based on the decomposition of the kernels of their iterates. Using this decomposition, we will show that regular operators and quasi regular operators satisfy, respectively, the strong and the weak Almansi decomposition properties.

Keywords: Regular operators, relatively regular operators, power relatively regular, quasi regular operators, generalized inverse, Moore-Penrose inverse, partial isometries, power partial isometries, generalized range, Almansi-type decomposition.
2020 Mathematics Subject Classification: Primary 47A05, Secondary 47B37.

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Complement of operators type Fredholm on Banach lattices and their modulus

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Abstract: This paper studies certain aspects of domination of Fredholm operators on Banach lattices. We focus on the case of an operator T satisfying $id_E \leq T$, where id_E denotes the identity operator on a Banach lattice E, and our primary goal is to look for conditions under which T is a Fredholm operator. Additionally, we apply the findings to the modulus of a regular operator and look for conditions that ensure Fredholm's properties are inherited from T (or $|T|$) to $|T|$ (or T).

Keywords: Upper semi-Fredholm operators, Fredholm operators, Banach lattice, positive operator, order bounded operator, modulus of an operator.
2010 Mathematics Subject Classification: 46B42, 47B65, 47B60.

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On the A-contractions operators

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Abstract: The class of operators T acting on a hilbert space $\mathcal{H}$ relative to a positive operator A on $\mathcal{H}$ and satisfying $T^*AT \leq A$ is called A-contractions that generalizes the well-known contractions, one of the most studied and understood classes of operators, we'll try to take it as the starting point for our study.
Even there are differences between the two classes, our work on A-contractions operators is based on extending some properties of contractions. As it was possible for the classical Nagy-Foiaş-Langer and von Neumann-Wold decompositions, we will try to give the generalisation of the *Wold-Type decomposition* for an A-contraction T, but this would not be possible without specifying the appropriate assumptions.

Keywords: Contractions, A-contractions, Isometry.
2020 Mathematics Subject Classification: Primary 47A15, 47A63, 47B20.

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The matrix-valued complex moment problem

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Abstract:
In this oral communication, we consider the matrix-valued truncated complex moment problem. We notice first that if a truncated complex matrix-valued sequence admits a representing measure, then it is the initial data of an infinite complex matrix-valued sequence verifying some suitable finite-dimensional property. We show that finite-dimensional completion of a truncated data provides a necessary and sufficient condition, and hence a solution, for the matrix-valued truncated complex moment problem. As a consequence, we obtain a matrix generalization of Curto-Fialkow's result on flat positive extensions of moment matrices.

Keywords: Matrix-valued moment problem, finite-dimensional sequences, positive matrix-valued sequences, representing measure.
2020 Mathematics Subject Classification: Primary 44A60; Secondary 47A57.

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Multiple-term improvements of Jensen's inequality for (p,h)-convex and (p,h)-log convex functions

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Abstract: In this paper, we present several new multiple-term improvements of Jensen's inequality for (p,h)-convex and (p,h)-log convex functions. As applications of our results, we present new bounds by employing means and Hölder type inequalities for the symmetric norms for τ-measurable operators. We make links between our findings and a number of well-known discoveries in the literature. These advancements refine classical inequalities and extend their reach to a broader class of functions. The inequalities obtained provide tighter bounds under weaker assumptions, particularly in operator settings. The use of means and operator inequalities offers novel insights into the structure of convexity in functional analysis. Our results demonstrate that the generalized convexity assumptions, when combined with multiple-term frameworks, yield stronger inequality estimates. Moreover, we explore the implications of our results in the context of noncommutative integration theory. Examples and comparisons are provided to illustrate the effectiveness and generality of the proposed inequalities.

Keywords: (p,h)-convex function, (p,h)-log-convex functions, weak sub-majorization, Jensen's inequality, scalar means.
2020 Mathematics Subject Classification: Primary 26D07.

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Maps commuting with the $\lambda-$Aluthge transform for the Lie Star Jordan product

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Abstract: Let H and K be complex separable Hilbert spaces, such that $dim(H) \geq 2$ and $B(H)$ be the algebra of bounded linear operators of H on itself. For every A, B in $B(H)$, we denote by $[A, B]_{\triangleright} = \frac{1}{2}(AB - B^*A)$ the Lie star Jordan product. For every λ in $[0, 1]$, we denote by $\Delta_\lambda(A)$ the λ-Aluthge transform of A. We show that a bijective map $\Phi : B(H) \longrightarrow B(K)$ satisfies the following condition for some $\lambda \in (0, 1)$, $\Delta_\lambda([\Phi(A), \Phi(B)]_{\triangleright}) = \Phi(\Delta_\lambda([A, B]_{\triangleright}))$, for all A, $B \in B(H)$, if and only if there exists a unitary or anti-unitary operator $U : H \longrightarrow K$, such that $\Phi(A) = UAU^*$, for all $A \in B(H)$.

Keywords: Non linear preservers, $\lambda-$Aluthge transform, Jordan product of operators, Hilbert spaces, Spectrum and Trace, Orthogonal projections.
2020 Mathematics Subject Classification: Primary 47A10, 47A25, 47A56.

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Bargmann integral transform on the two-sided Bargmann-Fock space

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Abstract: We introduce and study the properties of an integral transform of Bargmann type on the so-called two-sided Bargmann-Fock space. We also establish its connection to the two-sided Fourier transform.

Keywords: slice regular functions, two-sided Bargmann space, two-sided orthogonal basis, reproducing kernel, integral transform of Bargmann type, Fourier transform .
2020 Mathematics Subject Classification: Primary analysis, algebra, geometry.

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Putnam-Fuglede theorems and orthogonality of an elementary operator in C_p classes

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Abstract: Given Hilbert space commuting operators $T, S \in \mathcal{L}(H)$, such that T is w-hyponormal with $\ker T \subseteq \ker T^*$ and S is normal. Let $\phi_{T,S} \in \mathcal{L}(\mathcal{L}(H))$ be the elementary operator defined by $\phi_{T,S}(X) = TXS^* - SXT^*$. In this paper, we show firstly that (1) $\ker(\phi_{T,S} \mid C_p) \subset \ker(\phi_{T^*,S^*} \mid C_p)$. (2) The range of $\phi_{T,S} \mid C_p$ is orthogonal to the kernel of $\phi_{T,S} \mid C_p$ ($\mathcal{R}(\phi_{T,S} \mid C_p) \perp \ker(\phi_{T,S} \mid C_p)$) if and only if $\ker T \cap \ker S = \{0\}$. Secondly, we will extend these results to the elementary operator $\Phi \in \mathcal{L}(\mathcal{L}(H))$ defined by $\Phi(X) = AXD - CXB$ where $[A, C] = [B, D] = 0$. Related orthogonality results for the elementary operator Φ are also given.

Keywords: Elementary operator, Putnam-Fuglede property, Range-kernel orthogonality, Log-hyponormal.
2010 Mathematics Subject Classification: 47B47, 47A30, 47B15,47A63.

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Multiple Positive Solutions for Fractional Elliptic Problems with Multi-Critical Hardy-Littlewood-Sobolev Exponents

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Abstract: In this paper, we investigate the existence of multiple positive solutions to the following multi-critical elliptic problem

$$\begin{cases} (-\Delta)^s u = \lambda |u|^{p-2}u + \sum_{i=1}^{k} \left(|x|^{-(N-\alpha_i)} * |u|^{2^*_{s,i}} \right) |u|^{2^*_{s,i}-2}u \text{ in } \Omega, \\ u \in H_0^1(\Omega) \end{cases} \tag{1}$$

in connection with the topology of the bounded domain $\Omega \subset \mathbb{R}^N, N \geq 4$, where $\lambda > 0$, $2^*_{s,i} = \frac{N+\alpha_i}{N-2s}$ with $N-4 < \alpha_i < N, \quad i = 1, 2, \cdots, k$ are critical Hardy-Littlewood-Sobolev exponents and $2 < p < 2^*_s = \frac{2N}{N-2s}$. We show that there is $\lambda^* > 0$ such that if $0 < \lambda < \lambda^*$ problem (1) possesses at least $\text{cat}_\Omega(\Omega)$ positive solutions. We also study the existence and uniqueness of positive solutions for the limit problem of (1).

Keywords: Multi-critical problem · Multiple solutions · Elliptic equation.
2020 Mathematics Subject Classification: Primary 58F15, 58F17, 53C35.

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FPA-PROPERTY

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Abstract: Given Hilbert space operators $A, B \in \mathcal{L}(H)$. The pair (A, B) satisfies the Fuglede-Putnam-Aluthge property if $AT = TB$ and $T \in \mathcal{L}(H)$ implies $\tilde{A}T = T\tilde{B}$, where $\tilde{A}$ is the Aluthge transform of A. We prove that the class of pairs (A, B) that possess the Fuglede-Putnam-Aluthge property includes pairs of quasinormal operators, pairs of partial isometries with a normal square, the class of pairs (A, B) such that A and B^* are p-hyponormal or log-hyponormal, pairs (A, B) where A is a dominant operator and B^* is p-hyponormal or log-hyponormal, the class of pairs (A, B) for which A and B^* are w_*-hyponormal and all pairs of operators satisfying the Fuglede-Putnam property (FP-property). We also show that, (1) If A is invertible, then (A, B) has the FP-property implies that $(\tilde{A}, \tilde{B})$ has the FP-property. (2) If A and B are iw-hyponormal, then (A, B) has the FP-property if and only if $(\tilde{A}, \tilde{B})$ has it too. We give some classes of operators A and B for which $(\tilde{A}, \tilde{B})$ has the FP-property.

Keywords: Aluthge transform, Fuglede-Putnam property, quasinormal operator, partial isometry, w-hyponormal operator, Range-kernel orthogonality.
2020 Mathematics Subject Classification: $47B47$, $47A30$, $47A63$, $47B20$.

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On the Joint numerical radius of generalized spherical Aluthge Transforms of operators

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Abstract: In this work, we generalize and refine several operator inequalities involving the joint numerical radius and the joint operator norm of spherical Aluthge transform to generalized spherical Aluthge transforms. Moreover, we investigate the link between nontrivial joint invariant subspaces of the generalized spherical Aluthge transform and the original commuting $d-$tuples of bounded operators.

Keywords: Polar decomposition; generalized spherical Aluthge transform; joint numerical radius; joint operator norm.
2020 Mathematics Subject Classification: Primary 47A13, 47A12; Secondary 47A30, 47A15.

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On anisotropic double phase problems

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Abstract: We introduce a new class of anisotropic double phase equations with variable exponents. We establish several properties related to the anisotropic Musielak-Orlicz-Sobolev space associated with these equations, such as density results, continuous and compact embeddings. Furthermore, we establish the existence of at least one weak solution for our problem by using the surjectivity result for pseudomonotone operators. Additionally, under certain supplementary conditions on the nonlinearity, we derive the uniqueness of the solution.

Keywords: Anisotropic double phase operator with variable exponent, anisotropic Musielak-Orlicz-Sobolev space, continuous and compact embedding, density of smooth functions, local minimizer.
2020 Mathematics Subject Classification: 35D30, 35J62, 35P30, 46E35, 47H05.

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L-Dunford-Pettis property in Banach spaces

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Abstract: We introduce and study the concept of L-Dunford-Pettis sets and L-Dunford-Pettis property in Banach spaces. Next, we give a characterization of the L-Dunford-Pettis property with respect to some well-known geometric properties of Banach spaces. Finally, some complementability of operators on Banach spaces with the L-Dunford-Pettis property are also investigated.

Keywords: Dunford-Pettis set, Dunford-Pettis relatively compact property, Dunford- Pettis completely continuous operator.
2020 Mathematics Subject Classification: Primary 46A40, 46B40.

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On simple normal structure and best proximity points in reflexive Banach space

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Abstract: We introduce the concept of *simple normal structure* for a pair of subsets in a normed space that is not proximal. Using this concept, we show that if $\mathcal{E}$ is a reflexive Banach space, $\mathcal{A}$ and $\mathcal{B}$ are two nonempty, convex, bounded and closed subsets of $\mathcal{E}$ having a simple normal structure, and $\mathcal{T} : \mathcal{A} \cup \mathcal{B} \longrightarrow \mathcal{A} \cup \mathcal{B}$ is a cyclic relatively nonexpansive map, then $\mathcal{T}^2$ admits a fixed point in $\mathcal{A}$. Moreover, if $\mathcal{T}$ satisfies a min-max condition, then this fixed point of $\mathcal{T}^2$ is also a best proximity point for $\mathcal{T}$.
Using this concept, we obtain the same result for the best proximity point of a cyclic contraction map. We also provide an example of a reflexive Banach space that is strictly convex but not uniformly convex.

Keywords: Best proximity point; Fixed point; Cyclic contraction; Proximal normal structure.
2020 Mathematics Subject Classification: Primary 47H10, 47H09.

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About the numerical range of the basic elementary operator in semi-Hilbertian space

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Abstract: Let A be a positive bounded operator acting on a complex Hilbert space $\mathcal{H}$. For two bounded operators B and C on $\mathcal{H}$, we denote by $M_{2,B,C}$ the basic elementary operator on the class of Hilbert-Schmidt operators $\mathcal{C}_2(\mathcal{H})$, i.e., $M_{2,B,C}(X) = BXC$ for all $X \in \mathcal{C}_2(\mathcal{H})$. In this talk, we investigate the $\mathbb{A}$-numerical range $W_{\mathbb{A}}\left(M_{2,B,(C^{\sharp_A})^*}\right)$, where $\mathbb{A} = M_{2,A,A}$, $C^{\sharp_A}$ is the reduced solution of the equation $AX = C^*A$ and C^* is the adjoint of C. Within this framework, we show, under some A-hyponormality conditions, the following two equality:

$$\overline{W_{\mathbb{A}}\left(M_{2,B,(C^{\sharp_A})^*}\right)} = \mathrm{co}\left(\overline{W_A(B)} \cdot \overline{W_A(C)}\right),$$

where $W^A(\cdot)$ and $\mathrm{co}(\cdot)$ denote respectively the A-numerical range and the convex hull. Here, the bar stands for the closure. As a result of this equality, we establish that

$$\left\|M_{2,B,(C^{\sharp_A})^*}\right\|_{\mathbb{A}} = \|B\|_A\|C\|_A,$$

where $\|\cdot\|_{\mathbb{A}}$ and $\|\cdot\|_A$ designate the $\mathbb{A}$-operator seminorm and the A-operator seminorm, respectively.

Keywords: Semi-inner product; positive operator; hyponormal operator; numerical range; maximal numerical range; elementary operators.
2020 Mathematics Subject Classification: 47A12; 47B20; 47B65; 47B47; 46C05.

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On K-g-Fusion Frames within Hilbert C^*-Modules

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Abstract: Let $\mathcal{H}, \mathcal{H}_1, \mathcal{H}_2$ are Hilbert C*-modules over a fixed C*-algebra $\mathcal{A}$. The Hilbert C*-modules H generalize Hilbert spaces by replacing the field of complex numbers $\mathbb{C}$ with a C*-algebra $\mathcal{A}$. The inner product is extended to a mapping $\langle \cdot, \cdot \rangle$ from $H \times H$ to $\mathcal{A}$, satisfying certain properties analogous to those in a classical Hilbert space. The space $End^*_{\mathcal{A}}(\mathcal{H}_1, \mathcal{H}_2)$ stands for the set of all adjointable $\mathcal{A}$-linear operators from $\mathcal{H}_1$ to $\mathcal{H}_2$.
The aim of this talk is to extend results from the case to Hilbert spaces, particularly focusing on controlled K-g-frames. We examine the structure of these frames in Hilbert C*-modules and explore duality relations and perturbations

Keywords: Hilbert C^*-Modules, Frames, g-Fusion Frames, K-g-Fusion Frames, Controlled K-g-Fusion Frames.
2020 Mathematics Subject Classification: Primary 42C15; 46B15; 42C15 ; Secondary 46L05.

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Multiple Positive Solutions for Fractional Elliptic Problems with Multi-Critical Hardy-Littlewood-Sobolev Exponents

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Abstract: In this paper, we investigate the existence of multiple positive solutions to the following multi-critical elliptic problem

$$\begin{cases} (-\Delta)^s u = \lambda |u|^{p-2}u + \sum_{i=1}^{k} \left(|x|^{-(N-\alpha_i)} * |u|^{2^*_{s,i}} \right) |u|^{2^*_{s,i}-2}u \text{ in } \Omega, \\ u \in H_0^1(\Omega) \end{cases} \tag{1}$$

in connection with the topology of the bounded domain $\Omega \subset \mathbb{R}^N, N \geq 4$, where $\lambda > 0$, $2^*_{s,i} = \frac{N+\alpha_i}{N-2s}$ with $N-4 < \alpha_i < N, \quad i = 1, 2, \cdots, k$ are critical Hardy-Littlewood-Sobolev exponents and $2 < p < 2^*_s = \frac{2N}{N-2s}$. We show that there is $\lambda^* > 0$ such that if $0 < \lambda < \lambda^*$ problem (1) possesses at least $\text{cat}_\Omega(\Omega)$ positive solutions. We also study the existence and uniqueness of positive solutions for the limit problem of (1).

Keywords: Multi-critical problem · Multiple solutions · Elliptic equation.
2020 Mathematics Subject Classification: Primary 58F15, 58F17, 53C35.

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Multiple Solutions to the Fractional (p, q)-Laplacian Equations involving the critical exponents

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Abstract: In this article, we consider the following fractional (p, q)-Laplacian equation with critical exponent

$$(-\Delta_p)^{s_1} u + (-\Delta_q)^{s_2} u = \lambda g(x)|u|^{r-2}u + h(x)|u|^{p^*_{s_1}-2}u \text{ in } \mathbb{R}^N,$$

where $0 < s_2 < s_1 < 1$, $1 < q \leq p < r < p^*_{s_1}$ and $p^*_s := \frac{Np}{N-ps}$ for any $s \in (0, 1)$. Under certain assumptions on g et h, using an abstract critical point theorem from [3], we obtain a multiple solutions for λ sufficiently large. A similar problem with subcritical exponents is also considered.

Keywords: Fractional (p, q)-Laplacian, Variational methods, Concentration compactness principles, Palais-Smale condition, Critical point theorem.
2020 Mathematics Subject Classification: Primary 35J47, 35J50, 35J60, 35Q55, 35Q40

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Probability, Statistics and Modeling

On the xy models in mathematical epidemiology and their synergies with chemical reaction network

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Abstract: In this presentation, I will explore key concepts from Chemical Reaction Network (CRN) theory and Mathematical Epidemiology (ME), highlighting their intersection and mutual insights. The analysis focuses on the "xy models" which refers to compartemental epidemic models where x denotes the set of infected variables, and y denotes the set of uninfected variables. This investigation leads us to propose, for the first time, a formal definition of ME models, which I will introduce and which we also pinpointed in [1]. Additionally, I aim to raise awareness particularly among researchers outside the ME community of the utility of the next-generation matrix (NGM) approach for analyzing the stability of boundary equilibria , an aspect that remains underappreciated. Finally, I will showcase our Mathematica package, which facilitates these analyses, and which has been introduced in [2].

Keywords: mathematical epidemiology; biochemical interaction network; essentially non-negative ODE systems; symbolic computation; Routh-Hurwitz stability conditions
2020 Mathematics Subject Classification: 34A34; 92B05; 34D20; 68V35; 92E20.

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Linear Quantile Mixed Models: From Theory to Implementation

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Abstract: Dependent data arise in many studies, particularly under sampling designs such as multilevel, clustered, spatial, or longitudinal structures. Moreover, inference in quantile analysis has become an important tool to study effects across the full distribution of a response with complex dependencies. Geraci and Bottai [1] initially proposed a conditional quantile regression model for continuous responses to capture within-subject dependence. In this work, we focus on linear quantile mixed models (LQMMs) [2], an extension of quantile regression that adds random effects to account for dependence within clusters. The model uses the asymmetric Laplace distribution to enable maximum likelihood estimation of conditional quantiles while accounting for intra-cluster correlation. To reduce the computational burden and inefficiency, estimation is based on Gaussian quadrature and non-smooth optimization methods. Finally, these methods are implemented in the R package `lqmm` [3] to facilitate the practical application of LQMMs to real-world data.

Keywords: Random effects, Quantile regression, Linear mixed models, Hierarchical models, Best linear predictor, Asymmetric Laplace distribution
2020 Mathematics Subject Classification: Primary 62J12, 62G08, 62J05.

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Optimization of Subdivision Schemes for Geometric Curve and Surface Modeling Using Deep Learning

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Abstract: Subdivision schemes are essential tools in geometric modeling, enabling the generation of smooth curves, surfaces, and 3D shapes from discrete data. However, their performance and adaptability remain limited, especially in complex or non-uniform contexts. This work proposes an innovative approach that integrates artificial intelligence techniques, particularly deep neural networks, to improve the efficiency and accuracy of subdivision schemes. Relying on supervised learning, our method optimizes the prediction and adjustment of subdivision points in classical schemes such as Chaikins algorithm and cubic B-splines. The experimental results show a notable improvement in geometric accuracy and convergence, thus paving the way for more intelligent, adaptive, and high-performance modeling. This contribution aligns with current efforts to merge artificial intelligence and mathematical modeling for advanced geometric shape processing.

Keywords: Subdivision Schemes; Geometric Modeling; Artificial Intelligence; Deep Learning; Chaikins Algorithm; Cubic B-Splines; Shape Optimization; Data-Driven Modeling; Adaptive Geometry; Curve and Surface Processing.
2020 Mathematics Subject Classification: Primary 65D17; 68T07; 68U07

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From ARIMA to Machine Learning: A Comparative Study on Daily High-Speed Rail Demand Forecasting

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Abstract: Reliable short-term forecasting of high-speed rail (HSR) demand is essential for revenue management departments, enabling operators to adjust pricing strategies, manage seat inventory, and align resources with expected passenger volumes. In the context of Morocco's expanding HSR infrastructure, improving daily demand prediction is critical for enhancing operational efficiency and maximizing revenue potential. This study explores and compares the effectiveness of classical time series models and modern machine learning approaches for forecasting daily HSR demand, using historical daily data provided by the ONCF. The evaluation is conducted through standard error metrics to ensure robust model comparison. The findings demonstrate the advantages and limitations of each approach, offering insights into how data-driven forecasting can support more informed and agile decision making within the revenue management framework of Morocco's rail transport system.

Keywords: High Speed Rail (HSR), Demand Forecasting, Time Series Modeling, Machine Learning, ARIMA, Revenue Management, Transportation Modeling, Passenger Demand Prediction.
2020 Mathematics Subject Classification: Primary 62M10, 68T07, 90B25.

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A Mathematical Modeling Approach to the Co-Dynamics of Covid-19 and Tuberculosis

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Abstract: This study examines a mathematical model that captures the transmission dynamics of COVID-19 and tuberculosis within a population, emphasizing the interplay between the two diseases. The model incorporates key epidemiological factors, such as co-infection and disease progression, to better understand their joint impact. The existence and uniqueness of the endemic equilibrium are rigorously proven, followed by a comprehensive analysis of the equilibrium points and their local stability. To provide deeper insights, a sensitivity analysis is conducted to identify critical parameters influencing disease dynamics. Finally, numerical simulations are carried out to validate the theoretical results and explore potential scenarios, offering valuable perspectives for public health strategies

Keywords: COVID-19, Tuberculosis, Co-infection, Reproduction number.
2020 Mathematics Subject Classification: 92D30, 34A34, 34D23, 34C23 .

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Forecasting Morocco's Electricity Imports: A Comparative Analysis between SARIMA and SVR

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Abstract: This research focuses on the examination of Morocco's monthly electricity import data over a 14-year period, from 2008 to 2021. The primary aim is to examine and juxtapose the predictive performance of two distinct temporal modeling methodologies: the SARIMA (Seasonal Motion Integrated Autoregression) model, an advocated statistical technique, and the SVR (Support Vector Regression) model, a machine learning approach renowned for its ability to handle non-linear relationships in data. This study, through careful experimentation and rigorous evaluation using standard indicators of forecast accuracy, seeks to identify the model that offers the most reliable and accurate projections of Morocco's electricity imports. The study shows that the SVR model consistently outperforms the SARIMA model in terms of forecast accuracy, indicating that machine learning methods could provide significant improvements over conventional statistical models in the field of energy forecasting. Furthermore, the methodology and conclusions outlined in this research are not restricted to the case of electricity imports alone, they can be extended to the forecasting of other associated time series, such as electricity exports, and implemented on comparable energy datasets. This opens up new opportunities to improve energy planning and decision-making procedures through the use of sophisticated forecasting methods.

Keywords: SARIMA Model, SVR Model, Forecasting, Box Jenkins Approach, Morocco's Electricity Imports.
2020 Mathematics Subject Classification: Primary 62M10, 68T05, 62P30.

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Parameter estimation for reflected fractional Ornstein-Uhlenbeck processes with random effects

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Abstract: We consider a reflected Ornstein-Uhlenbeack processes driven by a fractional Brownian motion with Hurst parameter $H \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$, where the drift term depends on a random effect, the distribution of the random effect is characterized by unknown parameters to be estimated from continuous observations of the processes $\mathbf{X}$. In the cases where the random effect follows the Gaussian mixture distribution, we formulate the likelihood and we derive a parameter estimator, and we study their consistency. We illustrate the result by some numerical simulation using discrete observation of the processes.

Keywords: Reflected Fractional Brownian Motion, Reflected Ornstien Uhlenbeack process, Stochastic Differential Equations, Girsanov-type Formula, Random Effects, Maximum Likelihood Estimation.
2020 Mathematics Subject Classification: Primary 60H30, 62M09.

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Safety Distribution Analysis within Structured Epidemic Frameworks

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Abstract: We analyse conditions on a given a non-linear SEIRS epidemic model parameters, to provide a safety set for the epidemic containment, which means identifying an area in which the parameters of the epidemic remain under control and prevent an uncontrolled spread of the disease. After having determined this set, we look for control actions constraining the epidemic to remain within the safety set with infection rates below an allowed threshold. This means that for any initial state in a certain safety set of the state space, there exists an appropriate control strategy maintaining the state of the system in the considered safety set. Using advanced mathematical tools one can determine conditions for the stability of the epidemic and identify boundaries for the safety set. These borders delimit the values of the parameters where the epidemic remains under control and does not lead to a peak in propagation. To ensure the solvability under feedback control of our problem, sucient assumptions are derived in terms of linear inequalities on the input vectors at the vertices of a polytope. The results of this study are of great importance to policy makers and public health professionals, as they provide critical information on the management of outbreaks in limited populations. By identifying risk areas and critical parameters, these results could contribute to the implementation of preventive measures and targeted interventions to control the spread of infectious diseases in such populations.

Keywords: Epidemic process; discrete-time system; positivity; stability; feedback control

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Advancing Brain Tumor Segmentation with Deep Learning: Insights from the BraTS Africa dataset

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Abstract: Precision medicine depends on brain tumor segmentation because it enables precise diagnosis, individualized treatment planning, and efficient therapeutic progress tracking. The BraTS Africa dataset is used in this study to investigate the potential of sophisticated deep learning architectures, such as **U-Net**, **ResNet**, and **Deep U-Net** in recognizing and defining brain cancers in MRI pictures. The model's segmentation accuracy and dependability under various imaging settings are assessed using the Dice similarity coefficient, Jaccard index, precision, and recall. The results show the advantages and disadvantages of each model and provide insightful information for improving deep learning techniques to handle clinical problems in various healthcare environments.

Keywords: Brain tumor segmentation; deep learning architectures; U-Net; ResNet; Deep U-Net; medical image analysis; MRI; BraTS Africa dataset; tumor delineation; performance metrics; precision medicine.

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Comparing Quantile Regression Approaches: Simulation-Based Insights for Enhancing Fire Risk Prediction with Extreme Modeling

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Abstract: This study evaluates multiple quantile regression techniques applied to extreme values, emphasizing their strengths and limitations in different contexts. Some of these methods rely on Monte Carlo simulations to estimate the spectral measure, while others employ machine learning with neural networks and the generalized Pareto distribution. A simulation-based framework is employed to analyze the effectiveness of various approaches in modeling extreme dependence structures and predicting extreme quantiles. The methods are tested on fire risk data from the Fez-Meknes region, where a positive correlation is observed between rising maximum temperatures and the frequency of fires. By highlighting the comparative performance of these techniques, this work advocates for a hybrid strategy that combines the advantages of distinct approaches to improve both the accuracy and interpretability of forecasts for extreme phenomena.

Keywords: Spectral measure, Bivariate extreme value distribution, Neural Network, Extreme quantile.
2020 Mathematics Subject Classification: Primary 62G08, 62P05, 60G70.

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Regularized Tensor Factorization for Hyperspectral Image Completion

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Abstract: This work introduces an enhanced low-rank tensor completion model designed for hyperspectral image reconstruction. The objective is to accurately predict missing values from a limited set of observations. Specifically, we propose a tensor factorization approach that ensures low-rank representation while integrating a structured regularization term to preserve local spatial and spectral consistency. Our method factorizes the tensor through a mode-3 tensor-matrix product, where a low-dimensional tensor is coupled with a low-dimensional matrix. The regularization strategy combines piecewise smoothness properties with a spectral coherence constraint, improving the reconstruction quality. To solve the optimization problem, we employ a Proximal Alternating Minimization (PAM) algorithm. Extensive numerical experiments validate the efficiency of the proposed approach, demonstrating superior performance compared to existing tensor completion techniques based on both qualitative and quantitative assessments.

Keywords: Missing data, tensor factorization, regularization, Hyperspectral image reconstruction
2020 Mathematics Subject Classification: Mathematical modeling, Optimization.

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Real-Time Planning of Customized Bus Routes Using Multi-Agent Deep Reinforcement Learning

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Abstract:

Public transportation plays a significant role in mitigating traffic congestion, increasing mobility for the population, and accommodating sustainable urban development. Given these considerations, the need for responsive bus route procedures that align or respond to the real-time demands of passengers has arrived.

The proposed framework is able to address the high-dimensional (complex) decision-making problem in the urban transportation environment, while maintaining the demand for practicality. The framework organizes multiple agents (representing buses), that are capable of learning optimal routing policies in line with local observations and global objectives. The decentralization of the agents allows for improved scalability and flexibility of the agent-based system to respond to demand based on traffic patterns, and passenger demand and distributions.

The learning structure of the framework utilizes higher-level neural network architectures such as encoder-decoder models to improve learning efficiency. The ability to learn efficient representations is important, as information is often embedded within multi-dimensional modes and more complex time-dependent patterns within urban transportation data. Our approach will educate the agents on how to learn the associated patterns within transportation that relate to the nature of travel along different existing bus and passenger dynamics.

Additionally, the proposed framework will operationalize a multi-objective reward function designed to balance passenger convenience, amount of travel time the passenger experienced, and operational costs. Therefore, this framework is more facilitative of efficient and sustainable transportation services. The analyses show our approach achieved a significant reduction in travel time and reliable bus services, all while optimizing the use of bus fleet of or organization, suggesting our research may be a feasible alternative to existing centralized approaches to route planning.

Keywords: Deep reinforcement learning, Multi-agent system, Markov decision process, Encoder-decoder neural network, Policy gradient.

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Systematic literature review of stability and hyperstability of some functional equations

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Abstract: This paper presents a systematic review of the exploratory literature (SLR) on the study of stability and hyperstability of certain functional equations (SHFE). We used Scopus, ScienceDirect, and Web of Science to identify 175 papers through four search queries. Applying the PRISMA protocol, we narrowed this down to 134 papers for analysis with NVivo, and after further examination, 70 papers were selected. Our findings highlight that the key topics are stability, hyperstability, superstability, functional equations, spaces, and inequalities. Additionally, the predominant methodology used by authors for approximating functional equations is stability and hyperstability within Banach spaces.

Keywords: SLR, stability, hyperstability, functional equation.
2020 Mathematics Subject Classification: 39B52, 54E50, 39B82.

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Energy Harvesting in a Time-Delayed MathieuDuffing MEMS Device

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Abstract: This paper delves into the study of energy harvesting from periodic and quasi-periodic vibrations in a nonlinear MEMS (Micro-Electro-Mechanical Systems) device characterized by the presence of time delays. The device under consideration incorporates a delayed MathieuDuffing type oscillator coupled with a delayed piezoelectric mechanism. To analyze the system's behavior, we employ the method of multiple scales, allowing us to approximate the periodic response and determine the corresponding power output near the principal parametric resonance. The quasi-periodic response and the corresponding output power are determined and illustrated by numerical simulations in the regions where they exist. A significant focus of the investigation is the impact of time delays on the performance of energy harvesting. By adjusting the time delay parameters, we aim to identify the optimal conditions under which the device can efficiently harvest energy. The study reveals that, with the right combination of time delay parameters, there exists an optimal range of excitation frequencies beyond the resonance point where the energy harvested from quasi-periodic vibrations reaches its maximum potential. To support the analytical findings, extensive numerical simulations are conducted. These simulations confirm the theoretical predictions, demonstrating the validity of our approach and the potential for enhanced energy harvesting performance through careful tuning of time delay parameters. The results underscore the importance of time delay considerations in the design and optimization of MEMS devices for vibration-based energy harvesting applications.

Modeling and Analysis of two-strain epidemic model with time delays and nonlinear incidence rate in complex networks

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Abstract: Inspired by [1, 2, 3], we propose a novel two-strain SIR epidemic model on heterogeneous complex networks, incorporating a nonlinear incidence rate and two time delays. The model admits four types of equilibrium points: the disease-free equilibrium, the strain 1 endemic equilibrium, the strain 2 endemic equilibrium, and the coexistence (both strains endemic) equilibrium. We investigate the global stability of these equilibria using Lyapunov functions and LaSalle's invariance principle [4]. Two basic reproduction numbers R_1 and R_2 are derived. Finally, we conduct numerical simulations on scale-free networks to confirm our theoretical findings.

Keywords: Global stability, Complex network, nonlinear incidence

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Optimization of Agricultural Production in the MENA Region Under Resource Constraints and Water Stress

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Abstract: Agriculture in the MENA region faces increasing challenges related to resource scarcity, particularly water, capital, and agricultural inputs. Water stress, exacerbated by climate change, reduces agricultural productivity and necessitates a rational management of available resources. In a context where rainfall is becoming increasingly irregular, it is crucial to determine the optimal levels of production factor utilization (labor, capital, fertilizers, irrigation) while integrating economic and environmental constraints. A better allocation of resources is necessary to ensure sustainable agricultural production that is resilient to climate fluctuations.

This study aims to optimize agricultural production in the MENA region by considering resource constraints (budget and inputs) and water constraints (availability of irrigation water and rainfall variability). The objective is to assess the impact of water stress on agricultural production and to identify the optimal levels of resource utilization based on climatic and hydrological conditions. An approach combining mathematical optimization and econometric modeling is employed to calibrate the model parameters and ensure a better consideration of economic and environmental dynamics.

The study relies on a dual approach that combines mathematical optimization and econometrics. Agricultural production is represented by a Cobb-Douglas production function, maximized under several constraints. First, budgetary constraints are imposed, limiting expenditures allocated to inputs (capital, labor, fertilizers, irrigation). Then, resource constraints are introduced to reflect the limited availability of production factors. Finally, water constraints are integrated to account for the effects of water stress and rainfall on agricultural productivity.

Model optimization is performed using the Lagrange method, allowing for the determination of optimal resource allocation conditions based on the identified constraints. Econometrics is then used to calibrate the parameters of the optimization model. To this end, we use data covering the period 1961-2022 for 17 countries in the MENA region, adopting the Generalized Method of Moments (GMM). This method corrects for endogeneity and heteroskedasticity to obtain economically interpretable and statistically robust coefficients.

The study will identify the optimal levels of agricultural input utilization based on the specific water and economic constraints of the MENA region. It will contribute to measuring the impact of water stress and rainfall on agricultural productivity and irrigation efficiency. Additionally, it will provide recommendations on the optimal allocation of agricultural resources to enhance the sector's resilience to climate change and water shortages.

Keywords: Agricultural production; resource allocation; water stress; MENA region; optimization; econometric modeling.

2020 Mathematics Subject Classification: 90C90 ; 91B76 ; 62P20; 49J20; 91B74.

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Modeling Integer-Valued Time Series: Theory and Applications of INAR, INMA, and INARMA Processes

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Abstract:
Integer-valued time series models are essential for analyzing count data in fields like epidemiology, finance, and public health, where discrete observations exhibit complex temporal dependencies. This presentation explores the core frameworks: INAR (Integer-valued Autoregressive), INMA (Integer-valued Moving Average), and their hybrid INARMA models. Emphasis is placed on the role of thinning operators in maintaining the discreteness of the series, and on innovation distributions—such as Poisson and negative binomial—to effectively capture overdispersion and zero-inflation. We also discuss strategies for model identification and selection, leveraging statistical criteria (e.g., AIC, residual diagnostics). The framework is illustrated through a real-data application, covering the full pipeline from model specification to validation. The goal is to provide actionable insights for deploying these models in scenarios where traditional continuous-valued time series approaches fail, particularly when dealing with skewed, sparse, or zero-inflated integer data.

Keywords: integer-valued time series, INAR-INMA-INARMA models, thinning operators, overdispersion, zero-inflation, count data analysis, model validation.
2020 Mathematics Subject Classification: Primary 62M10, 60G10; Secondary 62H12, 62P99.

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A Hybrid Approach Combining K-Means Clustering and Machine Learning for Photovoltaic Power Prediction

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Abstract: Forecasting solar energy production with high precision is essential for enhancing photovoltaic system efficiency and ensuring grid stability. This research explores a novel hybrid methodology applied to the Solar Resource Atlas of Morocco dataset, which includes key variables such as Global Horizontal Irradiance (GHI), Direct Normal Irradiance (DNI), Global Tilted Irradiance (GTI), and temperature (TEMP). K-Means clustering is employed to identify distinct weather patterns based on temperature, enabling tailored predictive models (XGBoost, LSTM) for each cluster. Experimental results indicate that LSTM demonstrates superior performance in capturing temporal dependencies across different weather conditions, outperforming XGBoost. Model performance is assessed using RMSE, MAE, MAPE, and R^2, confirming the effectiveness of this hybrid approach over traditional forecasting techniques. This study highlights the benefits of integrating unsupervised clustering with deep learning to enhance solar energy forecasting accuracy across diverse climatic conditions.

Keywords: Hybrid Forecasting Approach, Time Series Analysis, Photovoltaic Power Prediction.
2020 Mathematics Subject Classification: 62M10, 68T07 , 62H30.

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Enhancing image denoising: A novel non-local anisotropic diffusion framework based on Caputo derivatives and Gaussian convolution for the Perona-Malik model

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Abstract: In the field of image restoration, denoising is considered one of the most important techniques. It is a preprocessing approach aims to refine image clarity and enhance its overall quality by effectively reducing noise present in the image. The aim is to obtain good-quality images from a version degraded by additive noise or convolutional noise that introduces blur. As a result, more advanced treatments can be performed on the resulting image. In order to remove Gaussian noise from input images, we propose the following methodology using a fractional differential equation in time-space based on Gaussian convolution, where the integer and fractional order derivatives of Caputo can be discretized using finite difference and L_1-approximations. Once the equation is solved numerically, the scheme is applied to grayscale digital images using the presented algorithm. The parameters must be optimized and adjusted. As a result of testing with natural images, we are able to successfully suppress the noise present in the images. Aside from that. Our model demonstrates strong visual quality, as verified by the calculation of indexes such as Peak Signal-To-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM). Our denoising technique demonstrates its effectiveness in mitigating noise present in both MRI and X-ray image

Keywords: Image processing, Denoising, Fractional Caputo derivative, Perona-Malik model, Fractional diffusion PDE, Gaussian noise
2020 Mathematics Subject Classification: Primary 94A08, 35R11, 65M06.

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Maximal Output Admissible Set for Linear Distributed Systems with an Application to Disturbance Rejection

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Abstract:
This paper investigates the maximal output set for a class of linear distributed systems with discrete output. This exploration is novel, as previous studies primarily focused on localized systems. We define an initial state as output admissible if its corresponding output satisfies specified constraints. This set of initial states termed the maximal output set (MOS), is shown to be nonempty, bounded, and characterized by a finite number of inequalities under suitable assumptions. In addition to theoretical characterization, we propose an algorithmic approach. To illustrate our framework, we provide a numerical example involving a parabolic system. Furthermore, we apply our results to address a disturbance rejection problem, aiming to design feedback controls that ensure the robustness of the system's output against disturbances.

Keywords: Maximal Output Admissible Set ; Linear Distributed Systems ; Disturbance Rejection.
2020 Mathematics Subject Classification: 93,34H05.

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A New Estimation Approach for Structural Equation Models

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Abstract: Structural Equation Modeling is a multivariate technique that analyzes causal relationships between latent variables and their measured indicators, forming a hypothesis on their specific pattern of relations. The estimation process is a key step in modeling, often performed with the classical and the popular BFGS algorithm. A new approach was recently introduced by El Hadri et al. in the context of Path Analysis models. This method has strong convergence properties that outperform the BFGS algorithm, offering a simple and effective alternative. In this paper, we extend this new procedure to Structural Equation Models while maintaining its strong convergence properties and superior efficiency compared to the classical method.

Keywords: Structural Equation Modeling, Implied covariance matrix, Finite iterative method, BFGS procedure
2020 Mathematics Subject Classification: Primary 62H12.

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Proactive Threat Detection: Unsupervised Clustering Approaches for Anomaly Detection in Cybersecurity

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Abstract: Given the ever-increasing potential for cyber threat risks, proactive anomaly detection is vital to cyber security enhancement. This research focuses on the application of unsupervised clustering algorithms for the detection of anomalies in network traffic data. Two main methods will be explored: K-Means, based on distance minimization, and DBSCAN, leveraging density-based clustering for outlier detection. These approaches will be applied to publicly available benchmark datasets representing real-world network traffic. We aim to evaluate and compare the effectiveness of these clustering algorithms in identifying anomalous network behaviors. Performance assessment are conducted using clustering validation techniques as well as statistical measures such as precision, recall, and F1-score. This study seeks to provide insights into the reliability, advantages, and disadvantages of unsupervised clustering approaches for anomaly detection, contributing to the development of more effective and scalable proactive cybersecurity defense systems.

Keywords: Proactive Threat Detection, Anomaly Detection, Unsupervised Clustering, Cybersecurity, Network Traffic Analysis.
2020 Mathematics Subject Classification: 62H30, 62P25, 68T07.

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Existence and optimal control of Hilfer fractional stochastic pantograph differential equations

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Abstract: In this article, the existence and optimal control of stochastic pantograph differential equation involving the Hilfer fractional derivative was investigated, in which a set of novel conditions were built to illustrate the existence via the Schaefer's fixed point theorem and uniqueness via the Banach fixed point theorem. The existence of optimal control pairs for the corresponding Lagrange control systems is then explored. Stochastic elements introduce randomness, capturing real-world unpredictability, while pantograph equations incorporate scaled past states. Moreover, an example was proposed to showcase the practical applicability of the theoretical results.

Keywords: Fractional stochastic differential equations, Hilfer fractional derivative, pantograph equation, fixed point theorem.
2020 Mathematics Subject Classification: 26A33, 34K50, 47H10, 93E20.

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Proposal for an Intelligent Architecture for Resilient Smart Cities

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Abstract: This paper introduces a novel decision-making architecture for resilient smart cities, focusing on energy demand management and the cybersecurity of urban infrastructures. The originality of this work lies in the dual-level integration of optimization and protection mechanisms within Smart Urban Buildings (SUBs). At the first level, we develop a demand-side management (DSM) strategy that leverages SCADA monitoring and load-shifting techniques to reduce peak electricity consumption while preserving user comfort. At the second level, we propose a resilient, AI-driven architecture capable of dynamically randomizing resource execution environments, thereby mitigating the impact of potential cyber-attacks. We formally define the underlying decision-making problem and present a hybrid approach combining physical modeling and data-driven learning
The whole document should not exceed one page.

Keywords: Smart Cities, Demand-Side Management, Resilient, Cybersecurity
2020 Mathematics Subject Classification: Primary 20F10, 68T01.

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Advancing Brain Tumor Segmentation with Deep Learning: Insights from the BraTS Africa dataset

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Abstract: Precision medicine depends on brain tumor segmentation because it enables precise diagnosis, individualized treatment planning, and efficient therapeutic progress tracking. The BraTS Africa dataset is used in this study to investigate the potential of sophisticated deep learning architectures, such as **U-Net**, **ResNet**, and **Deep U-Net** in recognizing and defining brain cancers in MRI pictures. The model's segmentation accuracy and dependability under various imaging settings are assessed using the Dice similarity coefficient, Jaccard index, precision, and recall. The results show the advantages and disadvantages of each model and provide insightful information for improving deep learning techniques to handle clinical problems in various healthcare environments.

Keywords: Brain tumor segmentation; deep learning architectures; U-Net; ResNet; Deep U-Net; medical image analysis; MRI; BraTS Africa dataset; tumor delineation; performance metrics; precision medicine.

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Uniqueness of solutions to quadratic BSDEs with locally Lipschitz generator

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Abstract: We study the uniqueness of solutions of backward stochastic differential equations (BSDEs), which generator verifies $|F(t,y,z)| \leq \alpha_t + \beta_t|y| + f(|y|)|z|^2$, where α_t, β_t are positive processes and the function f is positive, continuous and increasing. The uniqueness of solutions of such BSDEs is derived when F is locally Lipschitz. As a byproduct: we show the existence of viscosity solutions to the associated semilinear partial differential equations, which can contain nonlinearity that has quadratic growth in the gradient of the solution.

Keywords: Quadratic backward stochastic differential equations, uniqueness of solutions, Comparison theorem, partial differential equation, viscosity solution.
2020 Mathematics Subject Classification: 60H10, 60H20, 60H30, 91G10

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A Novel Method for Estimating Structural Equation Models

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Abstract: Structural Equation Modeling (SEM) combines regression, path analysis, and factor analysis to analyze complex relationships between observed and latent variables. Existing estimation methods suffer from overfitting, high sensitivity to model specification, and slow convergence. We introduce a novel estimation approach based on a modified procedure that stabilizes numerical computation. The proposed method integrates a finite iterative refinement step inspired by recent advancements in recursive modeling. Simulations based on the Political Democracy dataset show improved convergence speed and reduced mean squared error. Compared to classical GLS and ULS methods, our estimator achieves better performance under model misspecification.
Keywords: Structural Equation Modeling, Estimation, Model Stability

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Blind Deconvolution Using Game Theory and CNNs on Grayscale Image Datasets

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Abstract: This work proposes an innovative approach to blind deconvolution by combining game theory with convolutional neural networks (CNNs), applied to grayscale image datasets such as MNIST and similar collections. The problem is modeled as a two-player game: **Player 1 (P1)** is a U-Net network, parameterized by θ, tasked with restoring a sharp image from its blurred observation, while **Player 2 (P2)** is a ResNet network, parameterized by ϕ, responsible for estimating the blur kernel. This model allows us to explore the interaction between image restoration and blur kernel estimation, within a framework where each network optimizes its own parameters while considering the objective of the other, as in a two-player game. The dynamic collaboration between these two networks enables joint optimization that significantly improves performance. Using datasets like MNIST, composed exclusively of grayscale images, we achieved impressive results in terms of image quality. In particular, we obtained an average PSNR of 38.69 dB on the last batch, with individual values ranging from 34.50 dB to 42.9 dB across different images. Finally, this method is compared to a classical approach without CNNs, clearly demonstrating the substantial advantages of using CNNs in the context of blind deconvolution.

Keywords: Blind deconvolution, Game theory, Convolutional neural networks, U-Net, ResNet, Blur kernel estimation, Grayscale images, MNIST, Image restoration.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

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Overcoming convergence problems in PLS Path modelling

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Abstract: Partial Least Squares Path Modeling (PLS-PM) is one of the methods most widely used in the Structural Equation Modeling (SEM). PLS-PM aims to study the relationships among several blocks of observed variables, usually called Manifest Variables (MVs), where each block is assumed to measure a construct defined as a latent variable. Hanafi (2007) points out that there are two procedures for calculating the latent variable scores : the original procedure as proposed by Wold, and extended by Hanafi (2007) called the Hanafi-Wold procedure (2020), and an alternative procedure introduced by Lohmoller called the Lohmoller procedure. The systematic use of the Lohmoller procedure fo computing the latent variable scores can be ineffective. The contribution of this article is to remedy the issue of non-convergence of the Lohmoller procedure. Consequently, a new procedure for computing the latent variable scores, called Signless Laplacian Matrix (SLM) will be introduced, the main difference between the two procedures (SLM and Lohmoller) lies in the use of two different matrices to perform their iterations, both monotony and error convergence for this new procedure will be established.

Keywords: Partial least squares path modelling, Lohmollerâs procedure Hanafi-Wold procedure, Laplacian matrix, Signless Laplacian matrix

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The impact of white noise and Lévy jumps on the dynamics of an SIR epidemic model

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Abstract: This work aims to investigate the dynamic behavior of a stochastic epidemic model that includes the immigration of both susceptible and infected individuals, along with random perturbations represented by white noise and Lévy jumps. Biologically, immigration represents the continuous inflow of individuals into the population, which can either fuel transmission chains or reintroduce the infection. First, we establish the existence, uniqueness, and positivity of the model's solution. Then sufficient conditions are presented for the extinction of the disease in terms of a threshold value R_{jump}, which generalizes the basic reproduction number under stochastic influences. We also prove that the number of infected individuals is always persistent in the mean. Numerical simulations are provided to illustrate how different stochastic effects influence the evolution of the epidemic.

Keywords: Stochastic model; immigration; extinction; persistence; white noise; Lévy jumps.
2020 Mathematics Subject Classification: 60G51, 60H10, 60H30, 92D25, 92D30..

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Blind deconvolution using bilateral total variation and nash equilibrium

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Abstract: Blind deconvolution is a fundamental problem in image restoration, where the goal is to recover a sharp image from a blurred version without knowing the blur kernel. This work presents a novel approach to blind image deconvolution using bilateral total variation (BTV) regularization, combined with Nash equilibrium methods. The BTV method effectively preserves edge information and reduces noise in the recovered image by leveraging the spatial and intensity relationships between neighboring pixels. Additionally, the Nash equilibrium framework is employed to model the interaction between the image restoration process and the underlying blur kernel, ensuring that the solution is optimal in a game-theoretic sense. We demonstrate the efficacy of our method through a series of experiments, showing improved performance in both restoration quality and computational efficiency compared to traditional approaches. The proposed approach offers a promising solution for blind image deconvolution tasks in various applications, such as image denoising, super-resolution, and medical imaging.

Keywords: Blind deconvolution; bilateral total variation; Nash game; deblurring image; optimization.
2020 Mathematics Subject Classification: 65J22, 94A08, 49Q20.

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Log-volatility models in presence of zero returns : log-GARCH or SV ?

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Abstract: In this work, we address the challenges posed by zero returns in both log-GARCH and stochastic volatility (SV) models, with a particular focus on their asymmetric variants. Building upon previous imputation techniques for handling zero returns, as discussed in [1,2, 3], we propose a unified approach that enhances parameter estimation accuracy for both model classes. Specifically, we employ the Quasi-Maximum Likelihood (QML) estimation method, incorporating the Kalman filter for both the asymmetric log- GARCH and asymmetric SV models, to ensure robust parameter estimation even in the presence of zero returns. A comparative study is conducted on a set of financial return series containing zeros, examining the performance of the models in question using our proposed estimation method.

Keywords: log-GARCH, SV, zero returns, Kalman filter
2020 Mathematics Subject Classification: Primary 62M10, 62P05, 91G70.

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Medical Image Segmentation with Dynamic Weighted Chan-Vese: Weight Prediction Using Gradient and CNN

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Abstract: Medical image segmentation, crucial for detecting brain anomalies, presents a major challenge due to texture variability and the presence of blurry edges. We propose an innovative approach based on the Weighted Chan-Vese model, where segmentation does not rely on fixed parameters but rather on adaptive weights that evolve based on the image content. These weights are predicted using gradient techniques and convolutional neural networks (CNN), offering a more flexible and precise approach.
The core idea of our method is to replace constant coefficients with dynamic weights calculated in real-time for each image, allowing for better capture of contours and regions of interest. Initially, the image is enhanced using a CLAHE algorithm to optimize contrast. Then, an initial segmentation is performed with the classic Chan-Vese, followed by refinement through weight prediction using the gradient method and a CNN model trained on annotated data.
The results demonstrate finer and more adaptive segmentation, particularly effective for complex images in medical exams, with significant potential for early detection clinical applications.

Keywords: Weighted Chan-Vese, Gradient, CNN, Medical Image Analysis, Segmentation, Early Detection.
2020 Mathematics Subject Classification: Primary 68U10, 68Q25, 68T45.

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Modeling Claim Costs and Premiums in Moroccan Auto Insurance with GLMs

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Abstract:
Despite economic challenges, Morocco's auto insurance sector is experiencing steady growth. To stay competitive, insurers must innovate and refine their pricing strategies to meet evolving customer expectations. While traditional linear models have been used to assess claim frequency and severity, they often fail to capture the complexities of real-world data.
This study applies the Generalized Linear Model (GLM) to more accurately model total claim costs and determine optimal premiums in an auto insurance portfolio. The approach includes data processing, segmentation of rating variables, and the selection of suitable statistical distributions using tools such as the Wald and deviance tests, implemented in SAS software.

Keywords: Auto Insurance, GLMs, Financial and Insurance Mathematics.
2020 Mathematics Subject Classification: 91Gxx, 62J12, 62P05.

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Stationary distribution of a stochastic epidemic model with logistic growth

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Abstract: In this paper, we investigate the long-term behavior of a stochastic epidemic model incorporating saturation incidence rates and logistic population growth. We begin by establishing the existence and uniqueness of a global positive solution. Subsequently, we construct suitable Lyapunov functions to derive the condition $R^s > 1$ for the existence of a stationary distribution, along with criteria for persistence in the mean. Additionally, we identify conditions, including $R^e < 1$, that lead to the exponential extinction of the infected population. Consequently, some examples and illustrative simulations are carried out to verify the main theoretical results.

Keywords: Stochastic epidemic model, Stationary distribution, Exponential extinction.
2020 Mathematics Subject Classification: Primary 60H10, 92D30, 60G10.

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Statistical Aspects of Black Holes

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Abstract: The radiation entropy of gravitational systems such as black holes has a geometrical construction given by extremal surfaces. However, due to the quantum characteristics of black holes, this surface can be generalized to quantum extremal surfaces. We outline the derivation of gravitational fine-grained entropy using the quantum version of extreme surfaces. We provide evidence about a new method to compute von Neumann entropy. We show the statistical aspects of these fine-grained entropy formulas that follow the Page curve depicted from a pure bulk standpoint using entanglement wedge reconstruction.
The whole document should not exceed one page.

Keywords: Black holes, Island entropy, Extreme surfaces.

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Regularized Generalized Canonical Correlation Analysis and Its Variants

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Abstract: This paper explores multiblock data analysis different methods, where the variables are separated into distinct blocks measured on the same individuals. We focus here on a special and powerful framework for exploring complex, high-dimensional and multicollinear datasets, called Regularized canonical correlation analysis (RGCCA) [1].
RGCCA is used to uncover relationships across different data blocks [2]. Still, it requires experts to define the links between them in advance [3], a step that can be challenging and could influence both the analysis and the results. In this paper, we aim to demonstrate the flexibility and practical utility of RGCCA through real-world applications, where we illustrate the flexibility and practical value of RGCCA, emphasizing its ability to provide stable and reliable insights while achieving strong goodness of fit [4].

Keywords: Multiblock datasets, RGCCA, Regularization, High-dimensional data.
2020 Mathematics Subject Classification: Primary 62J07, 62H20, 62H25.

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Optimal Control of Cardiovascular Diseases among HIV infected

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Abstract: In this paper, we formulated a new model of cardiovascular diseases as a complication of HIV infection and its HAART treatment with optimal control strategies. The stability of the equilibrium point for the model without control is established via the Routh Hurwitz criterion. In addition, our model is linked to three control measures, including the organization of programs to prevent HIV infection, awareness of the infected about the impact of HIV and HAART, and regular diagnosis and monitoring of risk factors of cardiovascular diseases in HIV-positive people. The objective function of optimal control problem aims to minimize the number of complications and associated costs. Then, we characterized the optimal controls by applying Pontryagin's Maximum Principle after having proven its existence. We used Gauss Siedel's iterative approaches to solve the resulting system. We used data of HIV from Burundi to estimate fixed and fitted parameters. We used HIV data from Burundi to estimate fixed and adjusted parameters, to be able to provide numerical simulations to illustrate our theoretical results. Ultimately, the controls reduce the risk of cardiovascular diseases and improve public health.

Keywords: Optimal Control, Mathematical Modeling, HIV, cardiovascular diseases, Stability.
2020 Mathematics Subject Classification: 49K15, 34D20.

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The reliability of the estimation of the GEV return level: A comparative study based on Spot Crude Oil Price: West Texas Intermediate

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Abstract: A lot of work has developed the estimated return level for linear normalization in the extreme value theory (EVT). We propose in this article a theoretical approach based on the von Mises distribution to estimate the return level for the EVT in the case of the exponential normalization. An numerical approaches based on an algorithm have been developed to estimate the extreme quantiles of GEV models in the case of linear and exponential normalization. The results show that the exponential normalization model (GEVE) provide more realistic estimations of return periods. The finding was highlighted in working with the Spot Crude Oil Price: West Texas Intermediate and the parameters are estimated by the maximum likelihood method..

Keywords: extreme value theory , linear normalization, exponential normalization, return period of pricing, extreme quantiles.
2020 Mathematics Subject Classification: Primary 62G30, 62J01, 62G15, 62L11.

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